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The use of the Penman Formula in Hydrology

By

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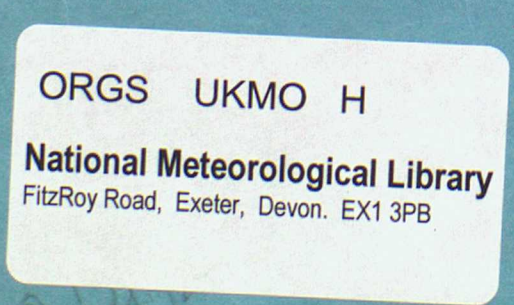

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The use of the Penman Formula in Hydrology

by B G Wales-Smith

This paper presents the results of investigations into the apparent validity of estimates of potential evaporation (hereafter normally referred to as PE) for periods of less than 30 days, making use of evaporation tank measurements for comparison.

Estimates of PE for Kew Observatory using 3 different measures of radiation (duration of bright sunshine, incoming radiation and net radiation) are compared.

A tentative suggestion is made for the combined use of Penman PE estimates and evaporation tank measurements in routine hydrology.

1.1. Data used and preparation of PE estimates

The Penman [1] formula requires, as input,

(i) a measure of solar radiation

(ii) saturation deficit of the air at screen level, air temperature and vapour pressure

(iii) run-of-wind at 2 metres above ground

The formula, a complete description of its terms and the meanings of the symbols used are set out in Appendix 1, together with notes on the use of different forms of data input and on a modification to take account of black body factor.

Data were obtained from 6 stations:

a. Kew Observatory, Surrey, (5 metres above mean sea level) is situated in low-lying parkland near the River Thames and to the west south west of central London. The area is surrounded by residential suburbs of west London.

b. Mildenhall, Suffolk, (5 metres above mean sea level) is in a generally agricultural area, near the River Lark and roughly mid-way between the Wash and the Thames Estuary.

c. Boscombe Down, Wiltshire, (126 metres above mean sea level) is in the open country of Salisbury Plain.

d. Thorney Island, Sussex, (3 metres above mean sea level) is close to the English Channel coast, east of Portsmouth and south of the South Downs.

e. Eskdalemuir Observatory, Dumfriesshire, (242 metres above mean sea level) is on a rising shoulder of open moorland in the upper part of the valley of the River White Esk in the southern uplands of Scotland. Hills rise to some 700 metres above sea level within 8 km to the north west.

f. Cardington, Bedfordshire, (28 metres above mean sea level), is in level farmlands in the valley of the Great Ouse some 3 miles south east of the small, industrial town of Bedford

The same data period, March 1968 to February 1969 was used for the first 4 stations. May-June 1969 to December 1970 for the last 2.

Estimates were made for individual days and for periods of 5, 10 and 30 days.

The forms of data were as follows:

(i) duration of bright sunshine (by Campbell-Stokes recorder) for all 6 stations and, in addition, totals of incoming solar and net radiation for Kew.

(ii) saturation deficit obtained from the average of daily screen temperatures at 03, 09, 15 and 21 hours GMT and the average of vapour pressures at the same times; temperatures and vapour pressures from the same observations.

(iii) wind run at 2 metres was approximated from the averages of mean hourly wind speeds measured from anemograph records and "reduced" from aneometer head height to 2 metres.

Totals of incoming solar radiation were converted to give approximations to net radiation using a regression equation obtained by P B Wright from data for Kew for 1968

$$R_n = 0.54\, R_c - 45$$

where R_n - net radiation, R_c = solar radiation } in mWh cm^{-2}

The calculations were made using the Meteorological Office KDF9 computer and a Metacode programme originally written by P B Wright and subsequently modified for this investigation by P Benwell. An entirely independent, Algol programme, written by A G Seaton, was used (i) to check the calculations, (ii) to provide some sunshine-based estimates for Kew using albedo .07 to represent open water (iii) to provide figures for the comparison of 2 ways of using incoming radiation data and (iv) to make PE estimates for Cardington and Eskdalemuir.

Albedo .25 was used for all the estimates of PE over grass.

1.2. <u>Comparisons of estimates using various measures of radiation</u>

Fig.1. compares PE 30-day estimates for Kew using the 3 different measures of solar radiation. P B Wright[2] obtained an almost identical diagram using the same year's data for Kew Observatory, but using the 0900 hr GMT vapour pressure as an approximation to the 24 hour average and the mean of the day's maximum and minimum temperature as an approximation to the average temperature. Only one year's data have been analysed and it would be unjustifiable to attempt to make firm deductions. Suffice it to say that all 3 measures of radiation led to the same basic diagram of the "march of evaporation". Estimates made using sunshine were generally higher than those using net radiation during the season of high evaporation and estimates in which net radiation was estimated from incoming radiation (by simple regression) gave lower values than those obtained with measured net radiation in June, July and August. Collingbourne[3] obtained the following regression equations from 1967 net and incoming radiation data from Kew Observatory:

June and July $R_n = 0.475\, R_c - 3$ Correlation Coeff 0.91

Winter months $R_n = -0.292\ R_c - 12$ Correlation Coeff. - 0.3

Whole year $R_n = 0.55\ R_c - 62$

Wright obtained the regression equation

$$R_n = 0.54\ R_c - 45$$ for the whole year

from 1968 data from Kew.
(Rn and Rc in units of $mWh.cm^{-2}$).

Net radiation measurements are not generally available for the production of regression constants or, for that matter, for direct use in the Penman calculation.

Appendix 1 suggests a tried method for the calculation of the outgoing long-wave radiation in the Penman formula from measurements of incoming radiation.

Fig.2. compares calendar month PE estimates made (a) with values of solar radiation and sunshine duration and (b) with approximations to net radiation obtained, by regression formula, from incoming radiation.
These estimates were made using Seaton's programme and Collingbourne's equation.

It will be seen that there are significant differences.

5-day estimates

5-day estimates (based on 5-day means of the variables) are compared in the two scatter diagrams of Fig.3. (a) compares estimates made with net radiation with those made with an estimate of net radiation obtained (by regression) from incoming radiation and (b) compares estimates made with net radiation with those made with sunshine.
It can be seen that regression lines could be drawn to both sets of data by eye, with confidence and that both would give almost a 1:1 relationship.
It is interesting to note that there is rather more scatter at higher rates of PE in (b) (net/sunshine) than in (a) (net/incoming) but that the winter estimates (low rates of PE) are closer to the regression line in (b) than in (a).

In Figs 4 and 5 the seasonal relationships are examined in detail. It can be seen (Fig.4) that estimates made using incoming radiation were generally higher than those using net radiation from mid-March to early May, equal to or lower than "net estimates" in June, July and early August and more above than below the "net estimates" in the colder months.

The sunshine-based estimates were generally higher than the net radiation based estimates except for those made in March and early April.(Fig.5)

1.3. Comparisons of PE estimates for various periods at four stations

Figure 6 compares PE 30-day estimates for Kew, Boscombe Down, Thorney Island and Mildenhall using duration of bright sunshine as the measure of radiation.

It will be assumed that an estimate of PE by Penman's method for a period of 30-days is generally held to be meaningful, (assuming good quality data are used.)

The first step in the comparisons was to compare

(i) The sum of 30 1-day estimates)
(ii) The sum of 6 5-day estimates) with the corresponding
(iii) The sum of 3 10-day estimates) 30-day estimate

for each of 12 30-day periods for all four stations, for sunshine-based calculations, and additionally at Kew to make two more 12-"month" comparisons for calculations based on net radiation and upon incoming radiation respectively.

The estimates for Mildenhall using sunshine as the measure of radiation are presented in histogram form as Figure 7. The diagram is self-explanatory.

The percentages, from working tables, are set out in Table 1. For each station-year a heavy line divides percentages < 110 from those $\geq 110\%$. Although the percentages (30 x 1-day, 6 x 5-day and 3 x 10-day of 30-day PE) reach absurdly high values, ∞ at Mildenhall in Dec 1968 when the 30-day PE estimate was zero, these percentages refer to small absolute values of potential evaporation.

Table 2 compares pairs of PE estimates as follows:

(a) 5 x 1-day with 1 x 5-day
(b) 2 x 5-day with 1 x 10-day
(c) 10 x 1-day with 1 x 10-day

giving percentages of pairs having given ranges of difference in m.m.

It will be noticed that the (a) and (c) comparisons for Kew (using all three measures of radiation) are much better than at the other three stations, even though very few differences anywhere exceed 1.5 mm. On the other hand, the (b) comparisons make little distinction between Kew and the other stations. A suggested explanation for this apparent improvement of (a) and (c) comparisons at Kew over those at the other stations is that the temperatures at Kew were measured in the North Wall screen and not in a conventional screen at standard height and with normal exposure. It is an established characteristic of the North Wall screen that minimum temperatures on "radiation nights" are normally considerably above those measured in the "garden" (or conventional) screen. (Garden screen temperatures were not available for 03, 09, 15 and 21 hrs GMT).

This means that on many days, the diurnal range of North Wall temperature is appreciably less than in the garden screen and hence the average North Wall

temperatures of any number of separate days are more likely to be closer to the average temperature over the whole period than are the normal screen daily averages to their period average if the daily and long-period averages are worked on temperatures at fixed times, one of which is not far from the normal time of night minimum temperature.

This argument gains some support from the lack of station distinction in the (b) comparisons in which no daily estimates are involved.

The temperature "damping" effect at Kew would operate through the $(e_s - e_d)$ section of the Penman aerodynamic term, which is, of course a multiplier for the whole term. The effect may also operate through σT_a^4 though this can not be argued strongly since this is not used in the net radiation form of the formula.

1.4. Consideration of Results

Estimates of (Penman) Potential Evaporation made from data from four well-separated stations having quite different topographical characteristics have been analysed.

(i) Evidence is produced which is in accordance with the general view that 30-day P.E. estimates made with Penman's formula and using good quality data present a coherent pattern of the annual march of evaporation using (a) various measures of radiation at one station and (b) using the same measure of radiation at several stations, at the same time.

(ii) It is shown that sums of PE estimates for several short periods approximate closely to PE estimates for equivalent longer periods, except in months with very low evaporation when the matter is of little importance.

(iii) It is shown that 5-day mean PE estimates using three different measures of radiation give basically similar results and that the scatter between sets of estimates may have a real seasonal bias.

2.1. Comparison of Penman P.E. estimates with tank evaporation measurements

2.1. Comparisons of tank evaporation measurements

Evaporation tanks (often called pans, especially when not sunk in the ground) have been in regular use in many parts of the world for a considerable time. W.M.O. Technical Note No.83 ("Measurement and estimation of evaporation and evapotranspiration" - A report of the Working Group on Evaporation Measurement of the Commission for Instruments and Methods of Observation) 1966, reported the results of some comparisons between various types of evaporimeters in several parts of the world.

Ratios (or "pan factors") have been produced relating measurements from various evaporimeters to one another and also to evaporation from large masses of fresh water.

Wright (1969)[4] found that mean values of evaporation measured by the Russian GG1 3000 tanks at Kew Observatory were, on average, about 10% higher than those measured by the British Standard Tanks. He gives the correlation coefficient between daily values from the two types of tank as 0.91 to 0.93 for the period June 1st to November 30th 1968.

Wales-Smith (1971)[5] analysed data from pairs of tanks* (one GG1 3000 and one British Standard) at Kew, Cardington and Eskdalemuir for the period 1 Oct.1969 to 30 Sept.1970. Mean daily values of evaporation lay close to regression lines at all three stations, but it appeared that whilst Wright's result (GG1 3000 evaporation > British Standard evaporation) was supported this may well be a characteristic of the Kew exposure and soil variation etc., since the opposite result was obtained at Eskdalemuir and Cardington. However when points for the 3 station-years were all plotted on one diagram the approximate regression line was very close to a 1:1 relationship. (Fig 8). (Brit =.027 + .932 Russ: R = .975: for values in mm)

Both tanks are of about the same depth (British 61 cm: Russian 60 cm at wall to 68.5 cm at centre of conical bottom). The (water) surface areas are, British square tank 33,445 cm^2, Russian circular tank 3000 cm^2. Thus the British tank has approximately 11 times the water surface area and volume of the Russian tank. All tanks were painted white.

Fig.9. shows daily evaporation measurements made at Kew during the (inclusive) period April to September 1969 with the two types of tank and using the same raingauge. Even for daily values, this analysis of 179 pairs for which J.S. Hopkins has calculated a correlation coefficient R = 0.887 and the regression equation.

Brit. = 0.078 + 0.890 Russ.

shows that for daily evaporation of up to more than 6mm most points lie within 0.5 mm of the regression line.

2.2. Consideration of Results

The drawbacks of sunken and above-ground tanks have been examined and described by a number of writers. It is known that there are daily differences between the readings of identical tanks thought to have identical exposuresat the same station. However the above and other comparisons suggest strongly that both types of tank (one 11 times larger than the other) are measuring the same thing on a daily basis and over longer periods.

*using rainfall measured at one gauge at each station.

For the purposes of the present study it has been accepted that, in a temperate climate, the measurements of (potential) evaporation made with a sunken tank of fresh water of the same order of size as the two types examined and a standard rain gauge with rim one foot above ground, both properly exposed, are somewhat coarse on a daily basis but meaningful if taken over a number of days (say ≥ 5).

3. Comparison of Penman PE Estimates and tank (Brit Std) measurements of evaporation.

3.1. 09-09 hr GMT tank evaporation and 00-00 hr GMT Penman estimates over a grassed surface.

This comparison is not a true one since tank evaporation occurs at all hours of the day and night on many days in the year. The overlap, 09 hr to midnight, does however, contain the period during which most incoming radiation is received.

Figs 10 to 12 show the comparison for estimates made using net radiation, incoming radiation and sunshine respectively. The scatter of points is considerable in each case and there seems little to choose between the three measures of radiation. J S Hopkins has calculated correlation coefficients and regression equations for sunshine and net radiation type estimates and it will be seen that for May-Sep 1969 at Kew, sunshine - based estimates were slightly better than those based on net radiation; there is no significant difference between the 3 correlations.

3.2. 5-day means of tank evaporation and 5-day estimates of PE.

Figs 13 to 16 show the comparison for estimates made with net radiation, incoming radiation, sunshine (with albedo 0.25 for grass) and sunshine (with albedo 0.07 for water.)

J S Hopkins has calculated correlation coefficients and regression equations for 13 to 15 and these show that all estimates are good approximations to tank evaporation. Although no calculation has been made, it can be seen that the correlation of Fig 16 is also good.

5-day means of tank evaporation and 5-day estimates of PE at Cardington and Eskdalemuir were also compared. The periods of comparisons were June 1969 to December 1970 at Eskdalemuir and May 1969 to December 1970 at Cardington. The comparisons were made between both British Standard and Russian GGI 3000 tanks and sunshine-based Penman estimates. The regression equations and correlation coefficients are given in the table below.

3.3. 10-day means of tank evaporation and 10-day estimates of PE.

Fig. 17 shows the good correlation between the estimates and measurements.

3.4. Daily means of 5-day estimates of PE calculated using sunshine and albedo 0.25 and of tank evaporation are plotted against time of year in Fig 18. The correlation coefficient and regression equation may be compared with those of Figs 10 and 12 to show the improvement obtained by taking pentad rather than daily values.

3.5. Consideration of results.

Using any of the three measures of radiation

(i) Daily estimates of PE and tank measurements are fairly well correlated.

(ii) 5-day estimates of PE and 5-day mean tank measurements are well correlated.

(iii) 10-day estimates of PE and 10-day mean tank measurements are very well correlated.

TABLE OF REGRESSIONS AND CORRELATIONS (a) Kew

Fig	Period	Comparison and No of days	Regression	R
10	1-day	PE (net radiation) vs T: May-Sep	P_N= .605 + .728T	.787
12	1-day	PE (sunshine) vs T: May-Sep	P_S= 1 + .604T	.799
18	5-day	PE (sunshine) vs T: May-Sep	P_S= .358 + .906T	. 94
15	5-day	PE (sunshine) vs T: 360 days	P_S= .102 + .954T	. 96
13	5-day	PE (Net radiation) vs T: 360 days	P_N= .028 + .958T	.954
14	5-day	PE (solar rad'n) vs T: 360 days	P_c= .165 + .890T	.952
17	10-day	PE (sunshine) vs T: 360 days	P_S= .039 + .999T	.971
			(b) Cardington.	
	5-day	PE (sunshine) vs BT: 585 days	P_s= .174 +.870 BT	.924
	5-day	PE (sunshine) vs RT: 585 days	P_s= .035 +.885 RT	.928
			(c) Eskdalemuir	
	5-day	PE (sunshine) vs BT: 545 days	P_s= .095 +.950 BT	.862
	5-day	PE (sunshine) vs RT: 545 days	P_s= .094+1.049 RT	.892

BT = British Standard Tank RT = Russian GGI 3000 Tank.

Conclusions

In Section 1 it was established that Penman PE estimates for periods of one or more days, using net radiation, incoming radiation or duration of bright sunshine, can be added together to give a close approximation to estimates made for longer periods except in (winter) months of low evaporation. (The trial covered one year at four stations in England).

In Section 2 it was established that two types of sunken evaporation tank, one having eleven times the water surface area and volume of the other, gave comparable results; the comparison being rather coarse on a daily basis but quite close for appreciably longer periods. This result at Kew is confirmed by the comparisons at Cardington and Eskdalemuir.

In Section 3 it was established that Penman PE estimates made over a grassed surface* using net radiation, incoming radiation or duration of bright sunshine gave results approximating more and more closely to tank evaporation with increasing length of period. 5-day estimates and 5-day tank evaporation are well correlated.

It is suggested, therefore, that Penman PE estimates made for ≥ 5 days and ≥ 5-day mean values of British Standard or Russian GGI 3000 tank evaporation are meaningful and comparable measures of evaporation and that estimates and measurements for shorter periods are coarse but still useful indications of the evaporative power of the atmosphere.

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* and sunshine-based estimates using albedo 0.07 for open water.

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4. <u>A suggestion for the combined use of Penman PE Estimates and Tank evaporation measurements</u>.

If it is accepted that a 5-day PE estimate, made with good quality data, is satisfactory and that a 24-hour tank evaporation measurement gives a coarse but still useful indication of PE and if it is only possible to make <u>consecutive</u> 5-day estimates, each estimate may be updated by means of tank evaporation measurements.

If $\overset{5}{\underset{1}{P}}$ denote the Penman estimate for day 1 to day 5, and so on, and T_1, T_2 are tank evaporation values we may form approximations as follows

$$\overset{5}{\underset{1}{P}} - T_1 + T_6 = \overset{6}{\underset{2}{P}} \quad \text{and so on to}$$

$$\overset{5}{\underset{1}{P}} - (T_1 + T_2 + T_3 + T_4 + T_5) + (T_6 + T_7 + T_8 + T_9 + T_{10}) = \overset{10}{\underset{6}{P}}$$

Fig 19, using data from Kew Observatory from 2 March 1968 to 28 August 1968, shows comparisons between day-by-day 5-day PE estimates and the first PE estimate (of a series) modified by subtracting one day's tank evaporation and adding one day's tank evaporation day by day.

If we define as a useful modification when the "tank method" tells us whether the current 5-day estimate is above, equal to or below the original 5-day estimate and as an incorrect modification when the tank method gives a value + for -, - for + or +/- for 0 change or 0 for +/-, we may mark the results as follows (out of 35 pentads).

Modification	1	2	3	4	5	
Number	$^{24}/_{35}$	$^{28}/_{35}$	$^{29}/_{35}$	$^{32}/_{35}$	$^{30}/_{35}$	correct

Fig 20 shows the comparison between consecutive 5-day PE estimates and approximations made by modifying the previous estimate by subtracting and adding 5-day tank evaporation totals.

Out of 71 attempts 57 were useful and 14 were incorrect. During the period May to September (inclusive) there were only 4 incorrect approximations.

Finally it may be of interest to examine an example of the same sort of exercise but, in this case, 1-day Penman estimates were used instead of tank evaporation values.

Fig 21 shows the result of this comparison for June 1968 at Mildenhall; 60% of attempts were useful and 40% were incorrect.

Appendix 1. Penman's (1948)[1] equation, using his original terminology and units is

$$E = \frac{\Delta}{\Delta + \gamma}\left\{ R_a(1-r)\left(a + b^{n}/_{N}\right)\right) - \frac{\Delta}{\Delta + \gamma}\left\{ \sigma T_a^{\,4}\,(0.56 - 0.092\sqrt{e_d}\,)(0.10 + 0.90^{n}/_{N})\right) + \frac{\gamma}{\Delta + \gamma}\left\{ 0.35\,(1 + \frac{U}{100})(e_a - e_d)\right)$$

E = Potential evaporation (mm/day)

Δ = de_a/dT_a, the slope of the saturation vapour pressure curve at air temperature.

γ = the constant of the wet and dry bulb hygrometer equation (in °F and mm of mercury $\gamma = 0.27$).

R = the Angot value of short wave radiation from sun and sky for a completely transparent atmosphere.

r = radiation reflexion coefficient or albedo.

a and b = the regression constants in the Aangstrøm equation relating incoming solar radiation to duration of bright sunshine.

n = measured duration of bright sunshine

N = possible maximum duration of bright sunshine

σT_a^4 = theoretical black body radiation at mean air temperature T

e_d = vapour pressure in mm. of mercury.

U = run of wind in miles per day at 2 metres

e_a = saturation vapour pressure (in mm of mercury) at temperature T_a.

T_2 = mean air temperature in degrees absolute

The first term is known as the Incoming short-wave radiation term and may be written

$$\frac{\Delta}{\Delta + \gamma} \quad R_c(1-r)$$

where R_c = measured short-wave radiation from sun and sky.

The second term is known as the outgoing long-wave radiation term. If measured values of duration of bright sunshine are not available but only measured values of R_c, if approximations to a and b can be made, we may write

$$^{n}/_{N} = \frac{1}{b}\left\{ \frac{R_c}{R} - a\right) \qquad \text{(Woodhead 1970}^{6}\text{)}$$

If measurements of net radiation are available we combine the first two terms and write

$$\frac{\Delta}{\Delta + \gamma} \quad R_n$$

Alternatively, an approximation to net radiation may be made from incoming radiation if sufficient values of both are available to produce a regression equation for the station or area.

In the computer programmes used for routine calculations and for this investigation, a black body correction was used (Budyko 1956)[7] (Grindley 1970)[8] to allow for vegetation not radiating as a perfect black body and we used $0.95\sigma T_a^4$

The third term is known as the aerodynamic term.
The programmes can accept various forms of input such as temperatures in Celsius, pressures in millibars and mean wind speed in knots at anemometer height for "conversion" to run of wind at 2 metres.

Appendix 2

Another use of tank evaporation values to supplement Penman estimates, successfully practised by several workers, is that of proportioning monthly PE estimates amongst shorter, component periods by reference to the tank data. Thus, if 60% of the tank total of evaporation occurred in the first 15 days of the month the same percentage of the PE estimate would be allocated to that period.

Appendix 3

Use of data from simple climatological stations, and use of fixed-hour data

Seaton and Wales-Smith [9] have investigated the effects on Penman estimates of PE of using approximations to the true averages of hourly values of temperature and vapour pressure.

a. Temperature An error of 1°C in the daily mean temperature in England in summer will lead to an error of 0.25 mm in the mean daily PE. Using the average of the temperatures at 03, 09, 15 and 21 GMT as an approximation to the average of 24 hourly values to provide 5-day mean temperatures (for a 4 station-year trial) gave daily mean errors in the month of greatest error of between .07 and .16 °C. For monthly estimates this would give errors of about 1.2 mm. (0.04 mm/day); a few pentad estimates could be in error by amounts exceeding 5%. Using the mean of the daily maximum and minimum temperatures as an approximation to the average of 24 hourly values gave errors of more than ¾°C on only 5% of occasions. Thus the ideal is the average of 24 hourly values but the 2 approximations thereto are acceptable.

b. Vapour pressure Approximating the average of 24 hourly values by using the average of the values at 03, 09, 15 and 21 GMT would, roughly speaking, give pentad PE errors exceeding 0.06 mm/day on only 1% of occasions. Using the vapour pressure at 09 GMT as an approximation to the average of 24 hourly values leads to larger errors than the 4-hour method. In the worst, summer months the monthly PE may be underestimated by 4 mm. Pentad errors are not too bad in summer but less acceptable in winter, but in winter months evaporation is small and unimportant by comparison with summer.

The methods of approximating the average of 24 hourly temperatures gave over-estimates of temperature on average and in the month of greatest error, leading to over-estimates of PE. The methods of approximating the average of 24 hourly vapour pressures also gave over-estimates, in the main, but these lead to under-estimates of PE.

For the period March 1968 to February 1969 2 sets of pentad PE estimates were compared for Kew and Boscombe Down. At each station both sets used the same wind and sunshine data but on the one hand the temperature and vapour pressure data were averages of values at 03, 09, 15 and 21 GMT, whilst on the other the temperatures were means of daily maxima and minima and the vapour pressures were for 09 GMT only. At Kew the correlation was .993 and at Boscombe .984.

Thus we see that the 2 methods of producing approximate averages give comparable results which, in turn, are quite good approximations to the PE estimates obtained from hourly data. Where possible, therefore, hourly data should be used and the estimates of PE derived from them should be given more weight in analysis than the useful but slightly lower status estimates derived from approximations, especially from the "max-min, 09 vapour pressure" method.

The accuracy of wind data used for PE estimates

The wind data used for Penman PE estimates are, generally, of 2 sorts. The first type being measured run of wind at 2 metres above the ground at variously "open" sites and the second being estimates of run of wind obtained from anemometer observations or from anemograph traces where the head of the instrument is exposed at 10 metres or above (as a rule). Attempts are usually made to "convert" the mean speeds to "equivalent speeds" at 2 metres, (mean speeds are multiplied by 24 to approximate to run of wind).

(i) Estimating average 24-hr wind speed from average of 4 observations

Using "spot winds" at 03, 09, 15 and 21 GMT at the station which showed the greatest variability in the analysis (Seaton [9]) there was only a one in a hundred chance that the error in the estimate of the daily mean wind speed over a pentad would exceed 1.5 kt which would result in a negligible error in PE.

(ii) Three sets of 2-metre wind stations in different parts of the country were chosen for the analysis, the component stations of the individual sets being only a few miles apart in each case. Although there were quite considerable differences, these were not so great as to result in important errors. To give an impression of the possible size of errors between a good, open site and an adjacent but very sheltered site in a typical summer month in England; if one station measured a wind run equivalent to a mean wind speed of 15 mph and the other measured a mean wind speed of 7.5 mph the resultant error in the Penman estimate could be 8%.

(iii) Comparisons were made between wind run actually measured at 2 metres above the ground at 3 stations and estimates made from anemographs at the same stations by using 24 X daily mean wind speed. The anemometer heads were at heights from 13 to 41 metres above ground and correction factors were applied to "reduce" the speeds to 2 metre equivalent speeds. Although the errors were considerable, they were not large enough to give PE errors of more than 2% for summer pentads as shown by Seaton [10].

Thus although the wind is not by any means a dominant factor in the penman equation it is still important enough to justify efforts to ensure that good and well-maintained instruments with similar "stopping speeds" should be exposed in positions where they will measure area-representative wind speeds.

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TABLE 1 Sums of 30 1-day estimates, 6 5-day estimates and 3 10-day estimates of P.E. expressed as percentage of the corresponding 30-day estimates. (PE values are given, in brackets, where percentages are ≥ 110%)

		MAR	APR	MAY	JUN	JUL	AUG	SEP	OCT	NOV	DEC	JAN	FEB
										(7.2)	(0.6)	(3.9)	(9.9)
KEW RN	1/30	103	105	102	99	101	101	101	105	147	433	208	134
	5/30	101	104	101	99	101	101	100	103	137	300	179	120
	10/30	102	103	101	99	100	101	99	102	115	250	141	104
										(11.7)	(2.7)	(5.4)	(18.3)
KEW RC	1/30	103	104	102	99	101	101	101	104	129	174	163	113
	5/30	102	103	101	100	101	101	100	103	123	144	143	108
	10/30	102	103	101	99	100	101	99	102	110	144	120	101
										(10.2)	(4.2)	(3.6)	(12.0)
KEW SUN	1/30	104	103	101	97	101	99	101	103	134	138	203	122
	5/30	103	103	101	99	101	99	101	103	128	121	181	113
	10/30	104	103	100	99	101	99	100	103	113	117	139	100
									(22.8)	(6.6)	(0)	(2.4)	(6.3)
MILDENHALL SUN	1/30	104	106	98	101	103	99	101	113	159	∞	192	149
	5/30	105	104	101	99	102	98	101	107	135	∞	183	127
	10/30	105	104	100	99	101	98	100	105	117	∞	150	106
									(21.3)	(6.9)	(2.4)	(2.4)	(8.7)
BOSCOMBE DOWN - SUN	1/30	101	104	97	97	99	101	98	111	164	258	254	130
	5/30	102	103	100	98	100	100	98	106	145	158	221	119
	10/30	102	104	100	98	100	100	100	105	119	146	150	99
									(28.2)	(8.7)	(5.1)	(3.3)	(10.2)
THORNEY IS. SUN	1/30	102	103	98	97	101	96	103	112	147	145	218	123
	5/30	103	103	100	98	101	99	101	106	135	123	179	115
	10/30	102	103	100	98	101	99	101	104	118	127	136	99

TABLE 2. Comparisons of pairs of P.E. estimates.

% of pairs within given ranges of difference

(a)

$\sum_{d=1}^{5}$ PE v.s. $\overline{PE}^{5day}$

		Difference in m.m. 0-0.5	0.6-1.0	1.1-1.5	1.6-2.0	2.1-2.5	2.6-3.0	3.1-3.5
KEW	RN	100						
KEW	RC	100						
KEW	SUN	99	1					
MILDENHALL	SUN	52	29	14	5			
BOSCOMBE D.	SUN	63	21	8	5	2		1
THORNEY IS.	SUN	53	28	14	5			

(b)

$\sum_{P=1}^{2}$ $\overline{PE}^{5day}$ v.s. $\overline{PE}^{10day}$

		Difference in m.m. 0-0.5	0.6-1.0	1.1 -1.5	1.6-2.0
KEW	RN	86	11	3	
KEW	RC	89	11		
KEW	SUN	86	14		
MILDENHALL	"	92	5		3
BOSCOMBE D.	"	83	11	6	
THORNEY IS.	"	86	11	3	

(c)

$\sum_{d=1}^{10}$ PE v.s. $\overline{PE}^{10day}$

		Difference in m.m. 0.-0.5	0.6-1.0	1.1-1.5	1.6-2.0	
KEW	RN	67	22	11		
KEW	RC	75	22	3		
KEW	SUN	72	22	6		
MILDENHALL	"	42	28	19	8	3
BOSCOMBE D	"	28	41	17	14	
THORNEY IS.	"	44	42	3	11	

Fig 1 Estimates of 30-day "month" totals of P.E. for Kew.

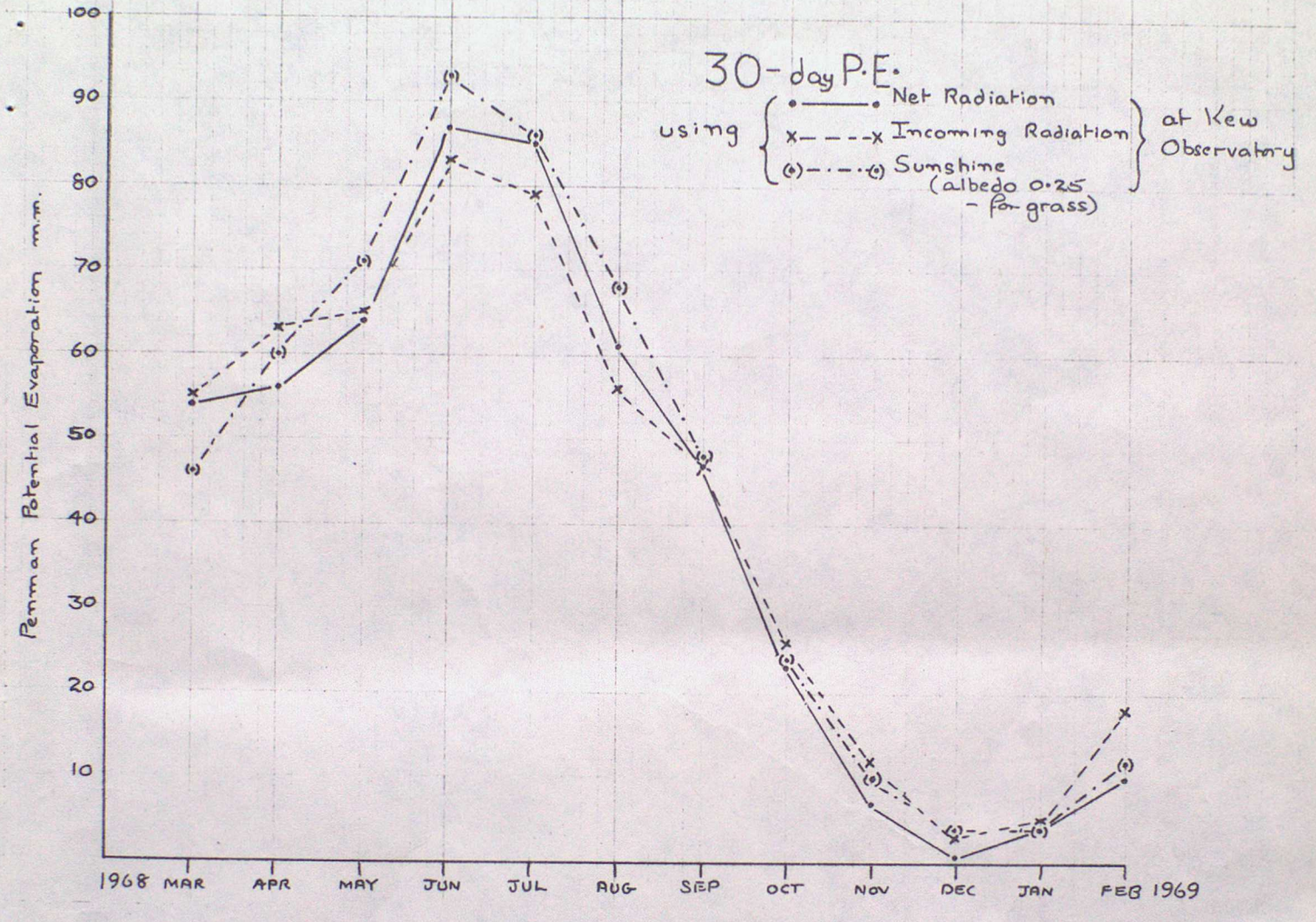


Fig 2. Comparison of calendar month P.E. estimates using solar radiation alone and with duration of bright sunshine. Kew. March 1968 - February 1969.

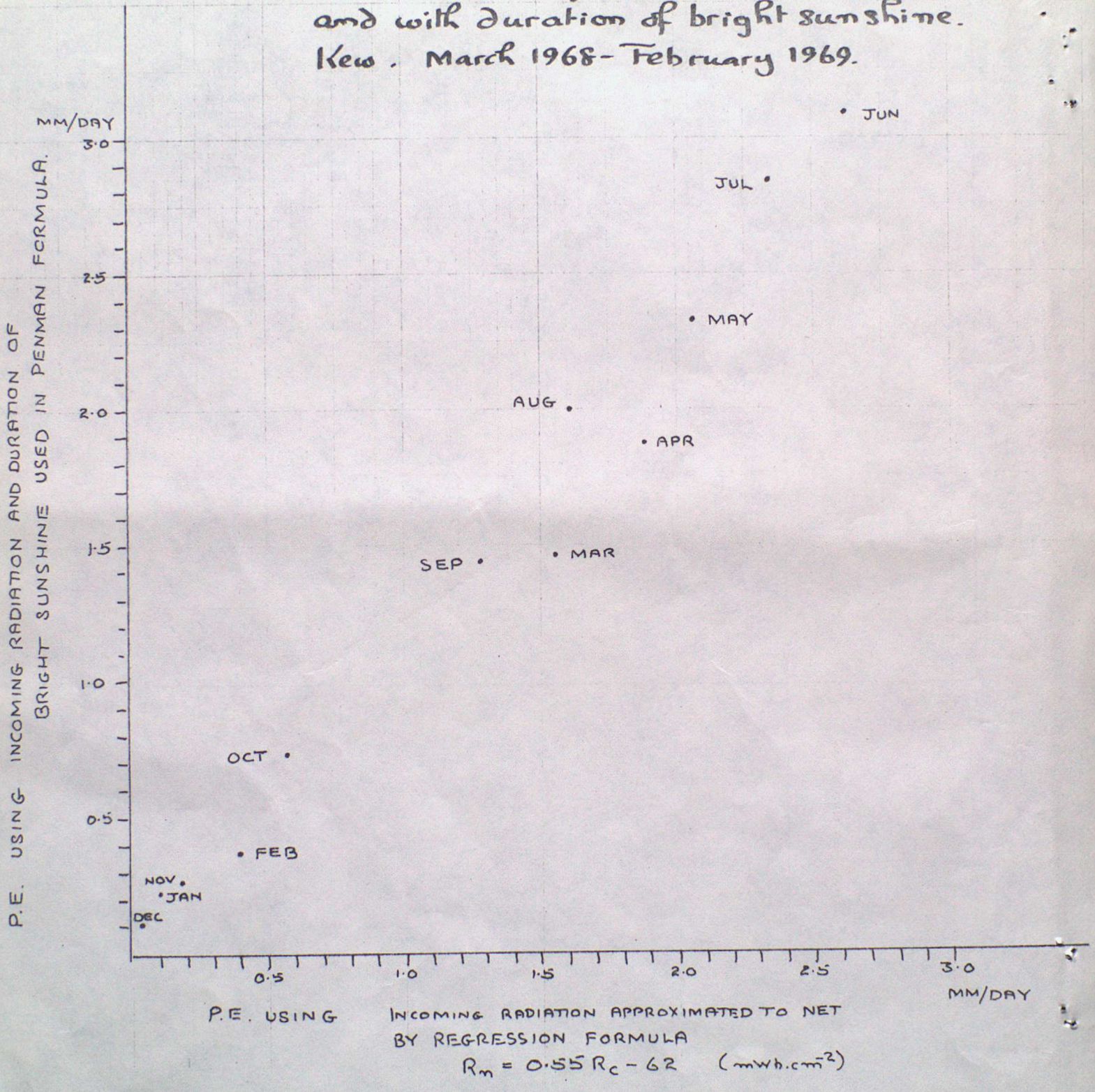
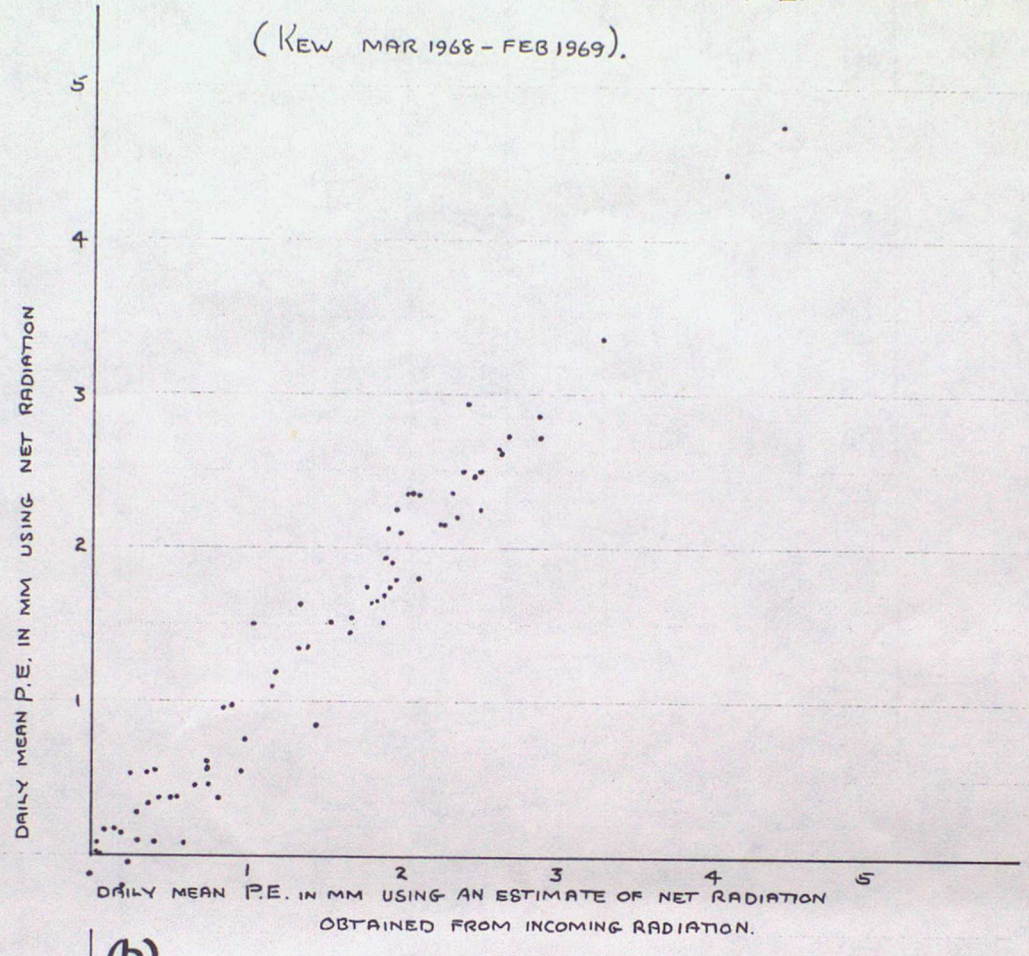


Fig. 3 (a) COMPARISON OF ESTIMATES OF P.E. FOR 5-DAY PERIODS (KEW MAR 1968 - FEB 1969).



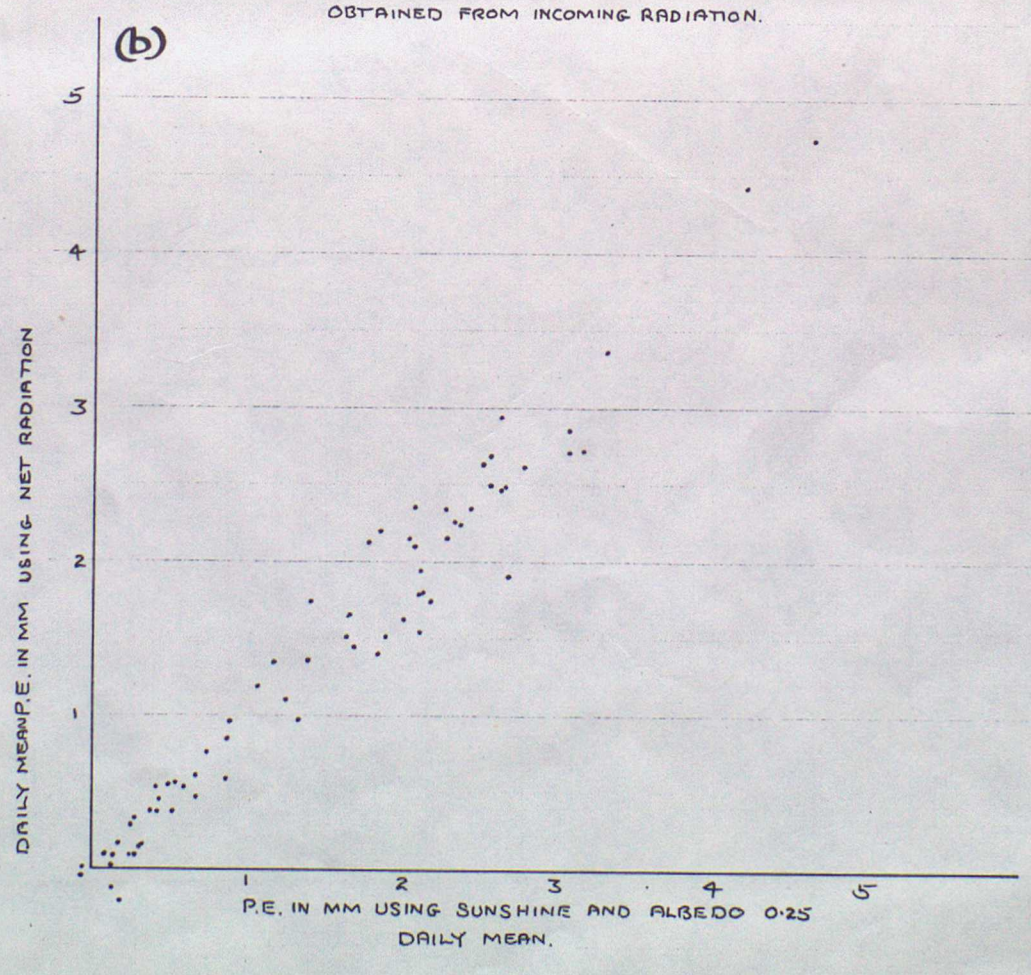


Figure 4

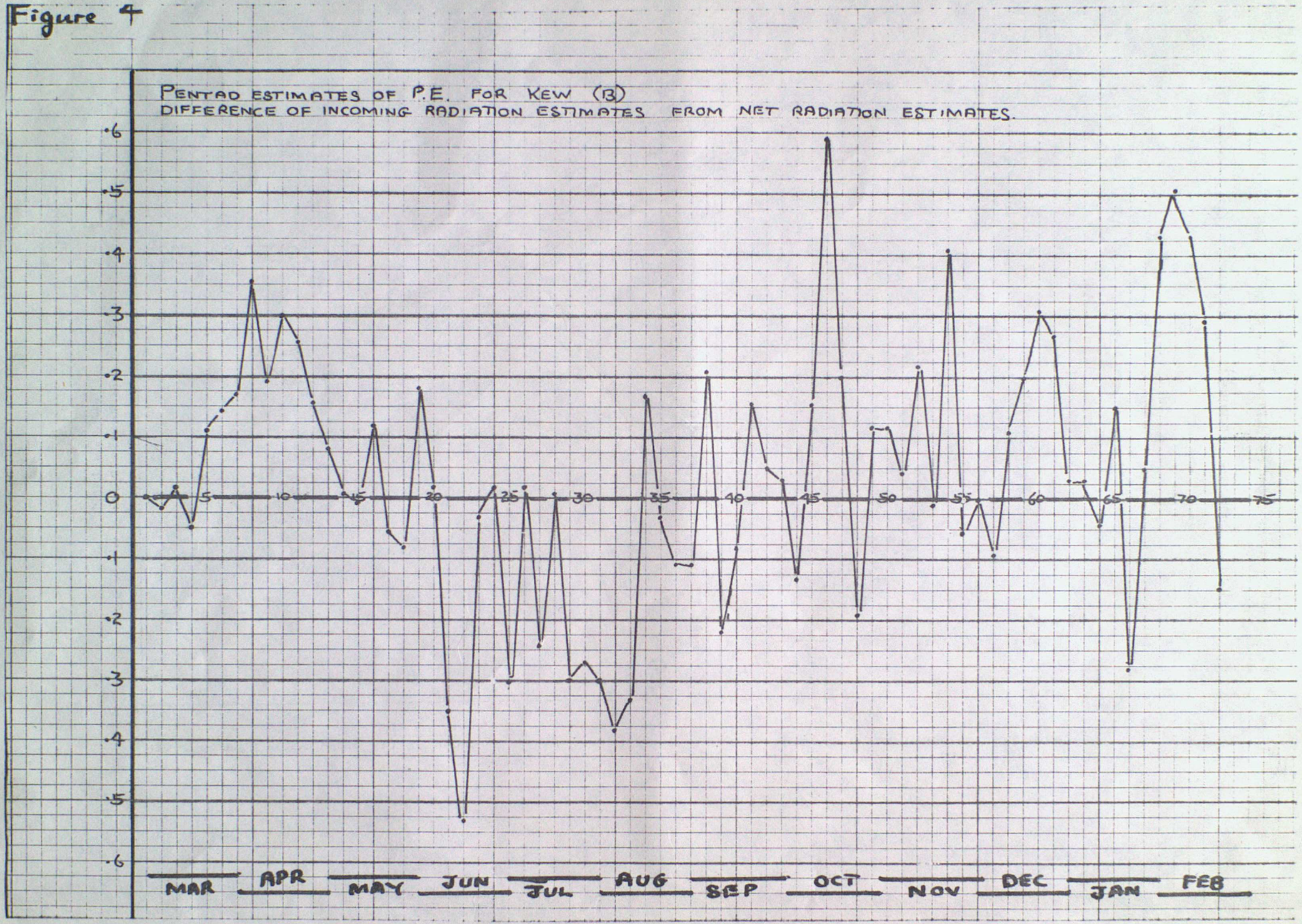

PENTAD ESTIMATES OF P.E. FOR KEW (B)
DIFFERENCE OF INCOMING RADIATION ESTIMATES FROM NET RADIATION ESTIMATES.
·6
·5
·4
·3
·2
·1
0
·1
·2
·3
·4
·5
·6
5
10
15
20
25
30
35
40
45
50
55
60
65
70
75
MAR
APR
MAY
JUN
JUL
AUG
SEP
OCT
NOV
DEC
JAN
FEB

Figure 5

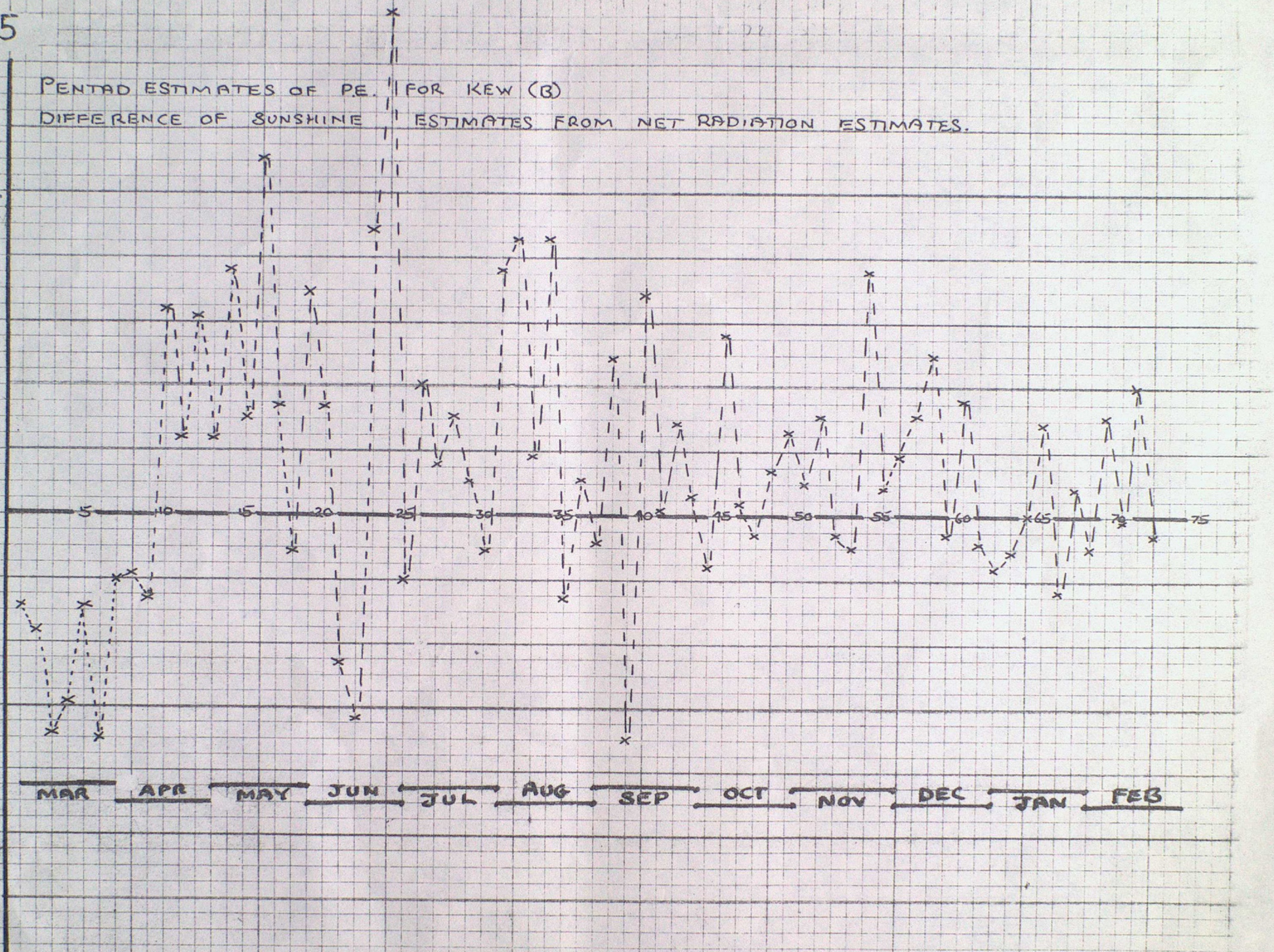

PENTAD ESTIMATES OF P.E. FOR KEW (B)
DIFFERENCE OF SUNSHINE ESTIMATES FROM NET RADIATION ESTIMATES.
·6
·5
·4
·3
·2
·1
0
·1
·2
·3
·4
·5
·6
5
10
15
20
25
30
35
40
45
50
55
60
65
70
75
MAR
APR
MAY
JUN
JUL
AUG
SEP
OCT
NOV
DEC
JAN
FEB

Fig 6 30-day PE estimates for four stations.

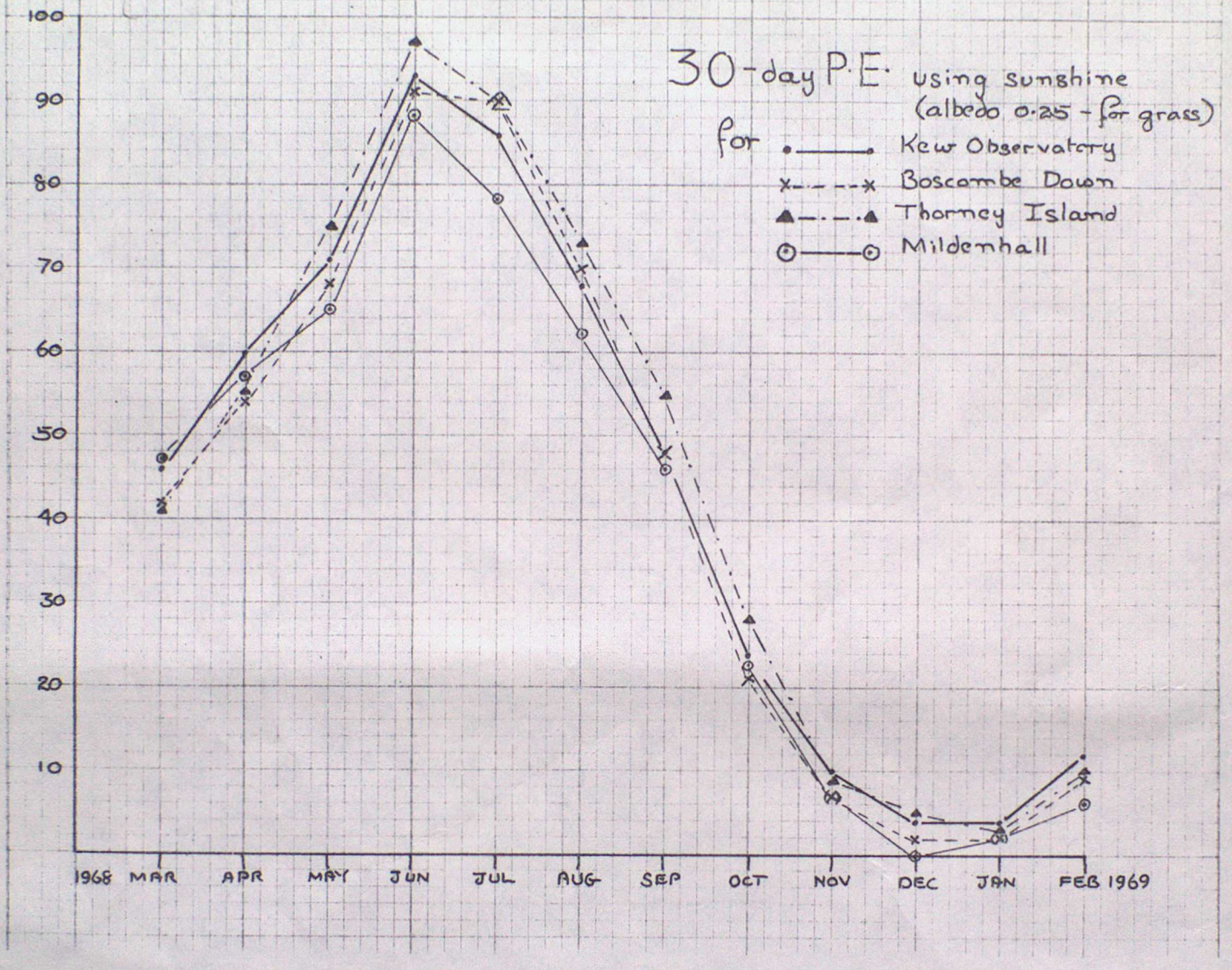


Fig 7

PENMAN ESTIMATES OF P.E. OVER GRASS AT MILDENHALL MAR. 1968 - FEB 1970

Cols. 30 – estimates from 30-day means of variables
1 – sets of 5-day sums of 1-day estimates
5 – estimates from 5-day means of variables
10 – estimates from 10-day means of variables

} arranged in chronological order starting at the base-line in each 30-day "month".

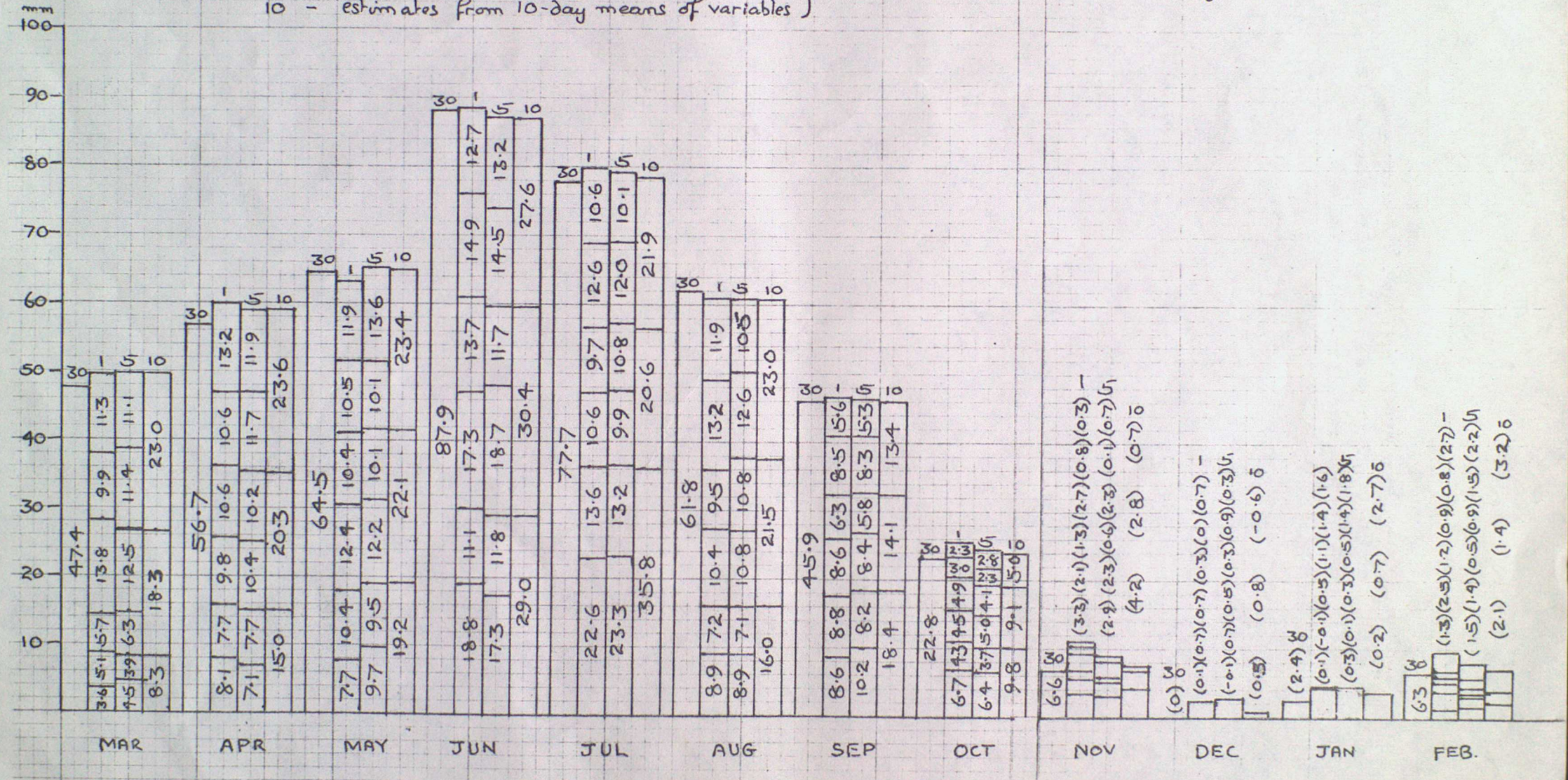


Fig 8

MEAN DAILY VALUES OF TANK EVAPORATION
AT KEW (K), CARDINGTON (C) AND ESKDALEMUIR (E)
FOR MONTHS OCT (10), NOV (11), DEC (12) 1969
AND JAN (1) TO SEP (9) 1970

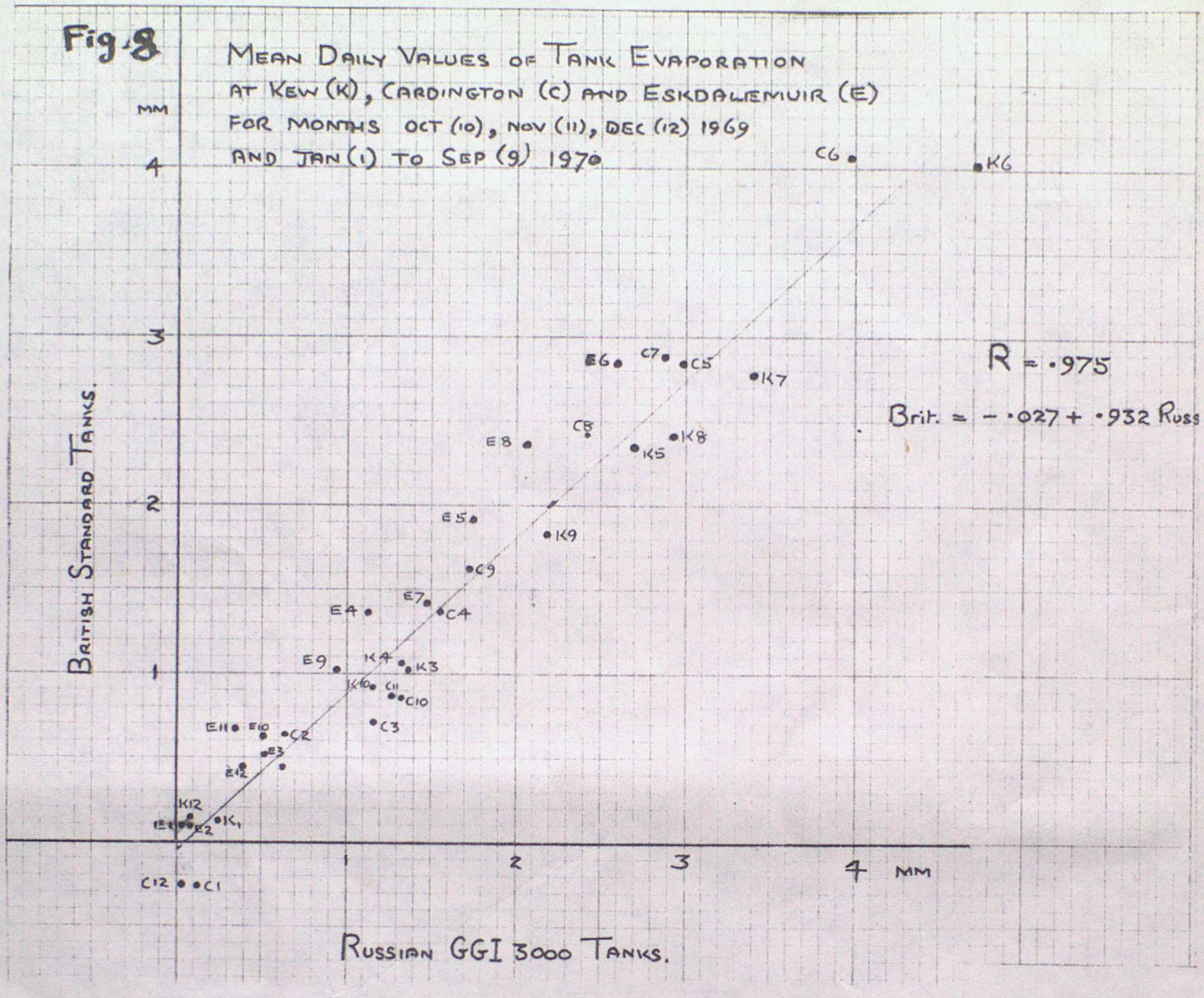
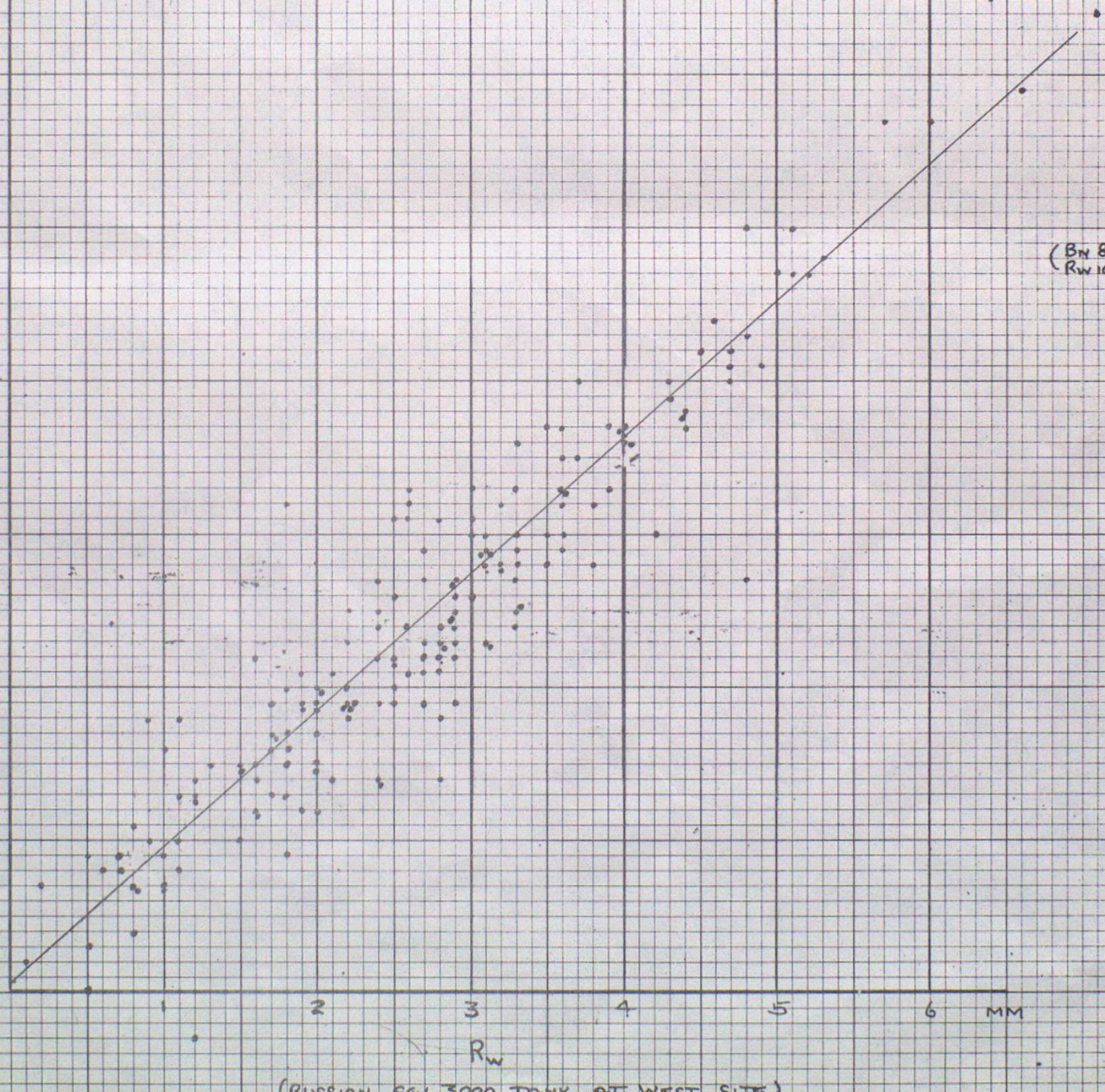


Figure 9

TANK EVAPORATION MEASUREMENTS (KEW)
APR-SEP 1969

Brit = + ·078 + ·890 Russ
R = ·887



(RUSSIAN G.G.I. 3000 TANK AT WEST SITE)

Figure 10

COMPARISON OF 09-09Z EVAPORATION TANK MEASUREMENTS AT KEW WITH 00-00Z PENMAN ESTIMATES OF POTENTIAL EVAPORATION OVER GRASS USING NET RADIATION MAY 1 1968 – SEP 30 1968

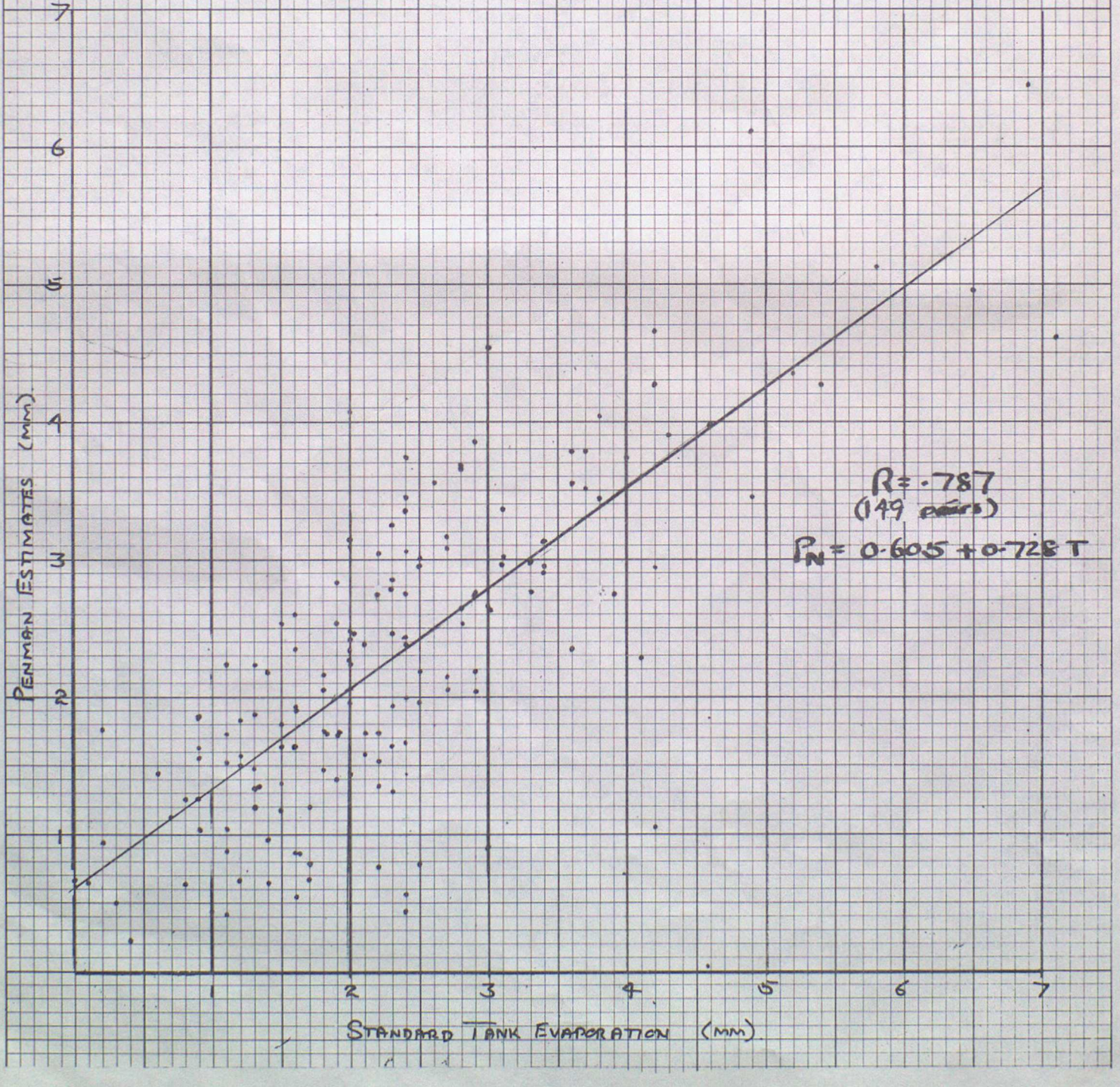


Figure 11

COMPARISON OF 09-09 Z TANK EVAPORATION MEASUREMENTS AT KEW WITH 00-00 Z PENMAN ESTIMATES OF POTENTIAL EVAPORATION OVER GRASS USING INCOMING RADIATION MAY 1 1968 - SEP 30 1968

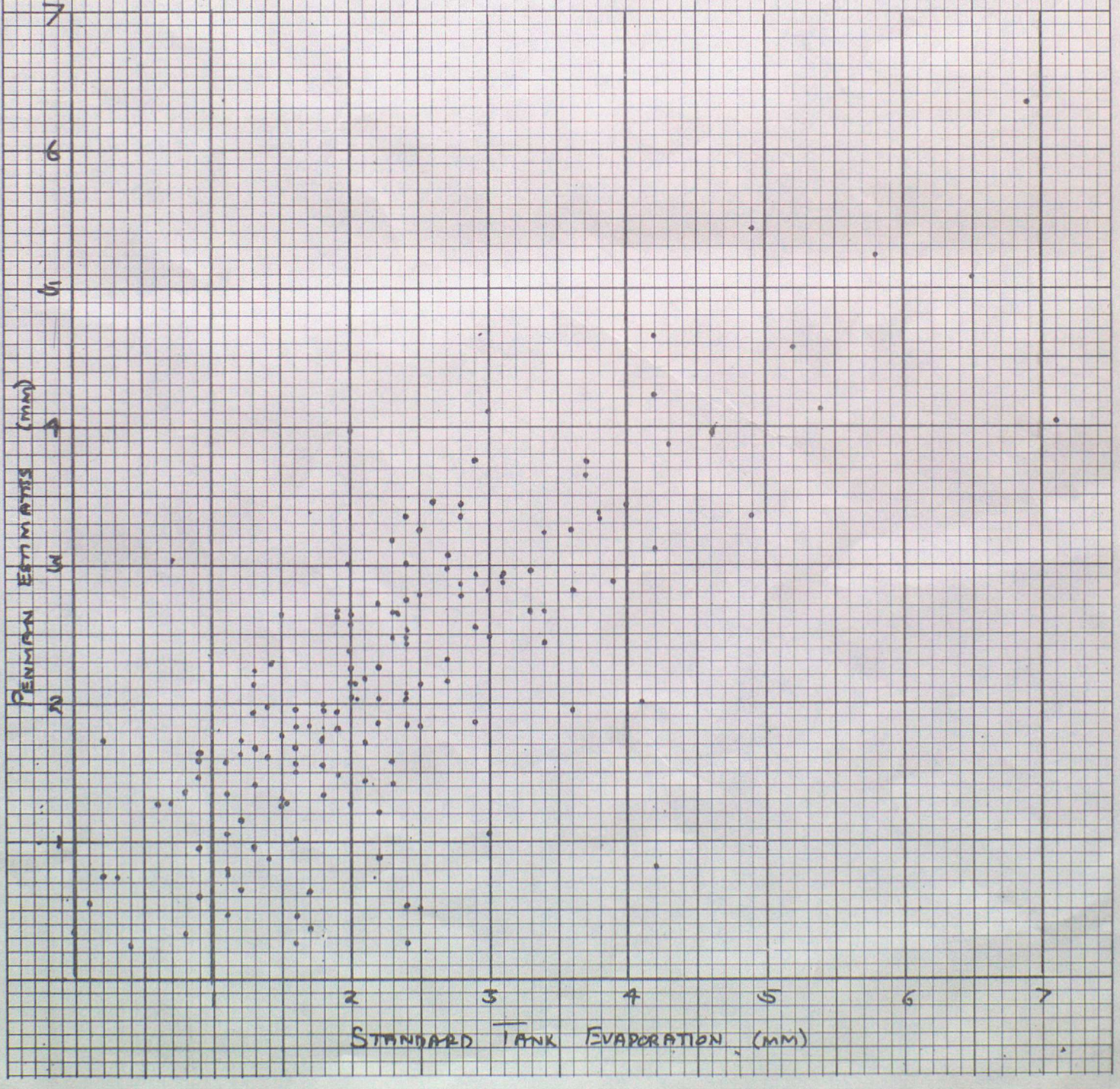


Figure 12

COMPARISON OF 09-09 Z EVAPORATION TANK MEASUREMENTS AT KEW WITH 00-00 Z PENMAN ESTIMATES OF POTENTIAL EVAPORATION OVER GRASS USING SUNSHINE MAY 1, 1968 – SEP 30, 1968.

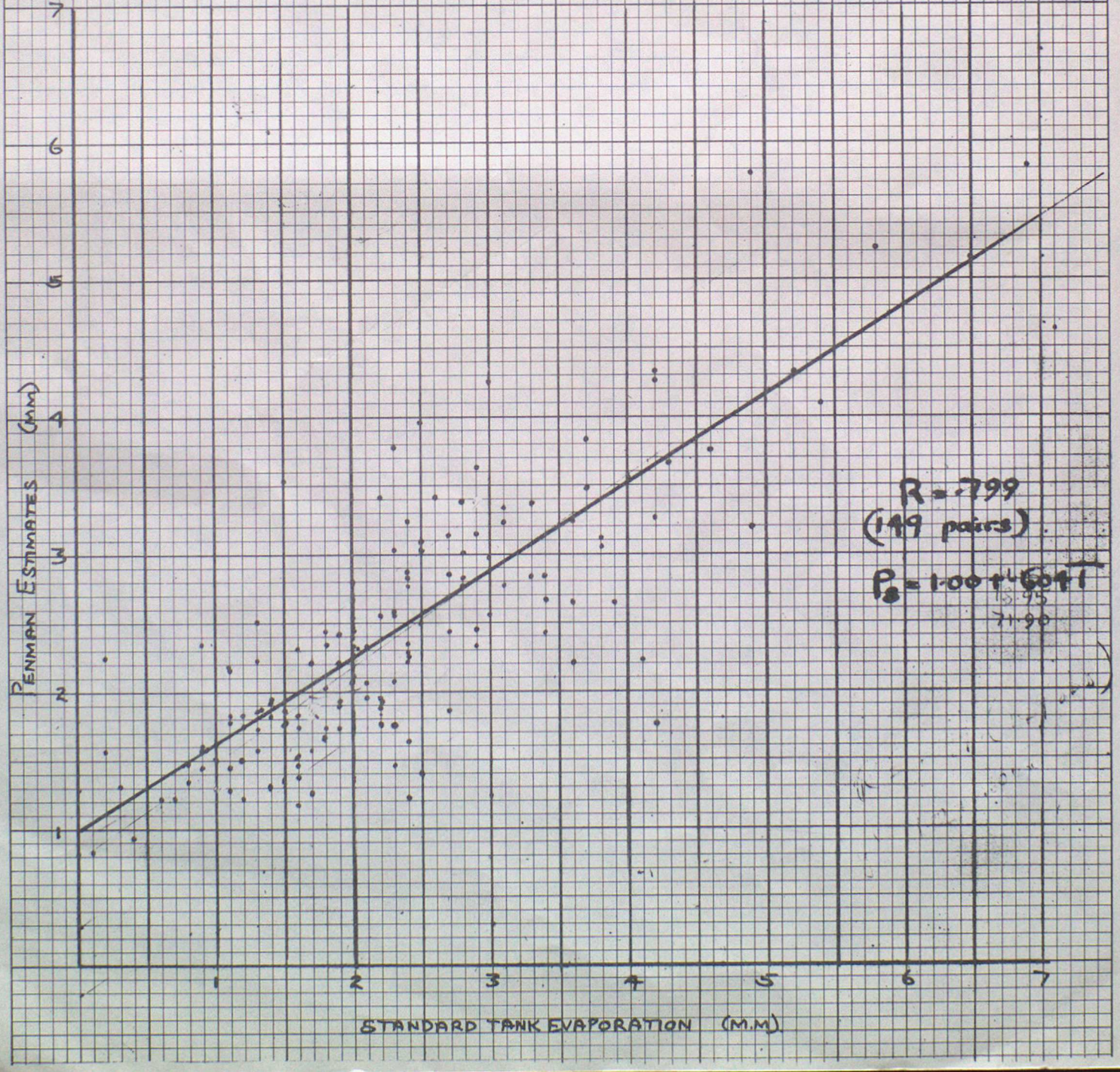


Figure 13

COMPARISON OF PENTAD DAILY MEANS OF

(A) EVAPORATION TANK MEASUREMENTS (09-09Z) } KEW MARCH 1968 - FEB 1969

(B) PENMAN ESTIMATES OF POTENTIAL EVAPORATION (00-00Z) }

OVER GRASS USING { NET RADIATION
SVP FROM MEANS OF 03, 09, 15, 21Z AIR TEMPS
VP " " " " VALUES
RUN OF WIND FROM MEAN DAILY WIND SPEED

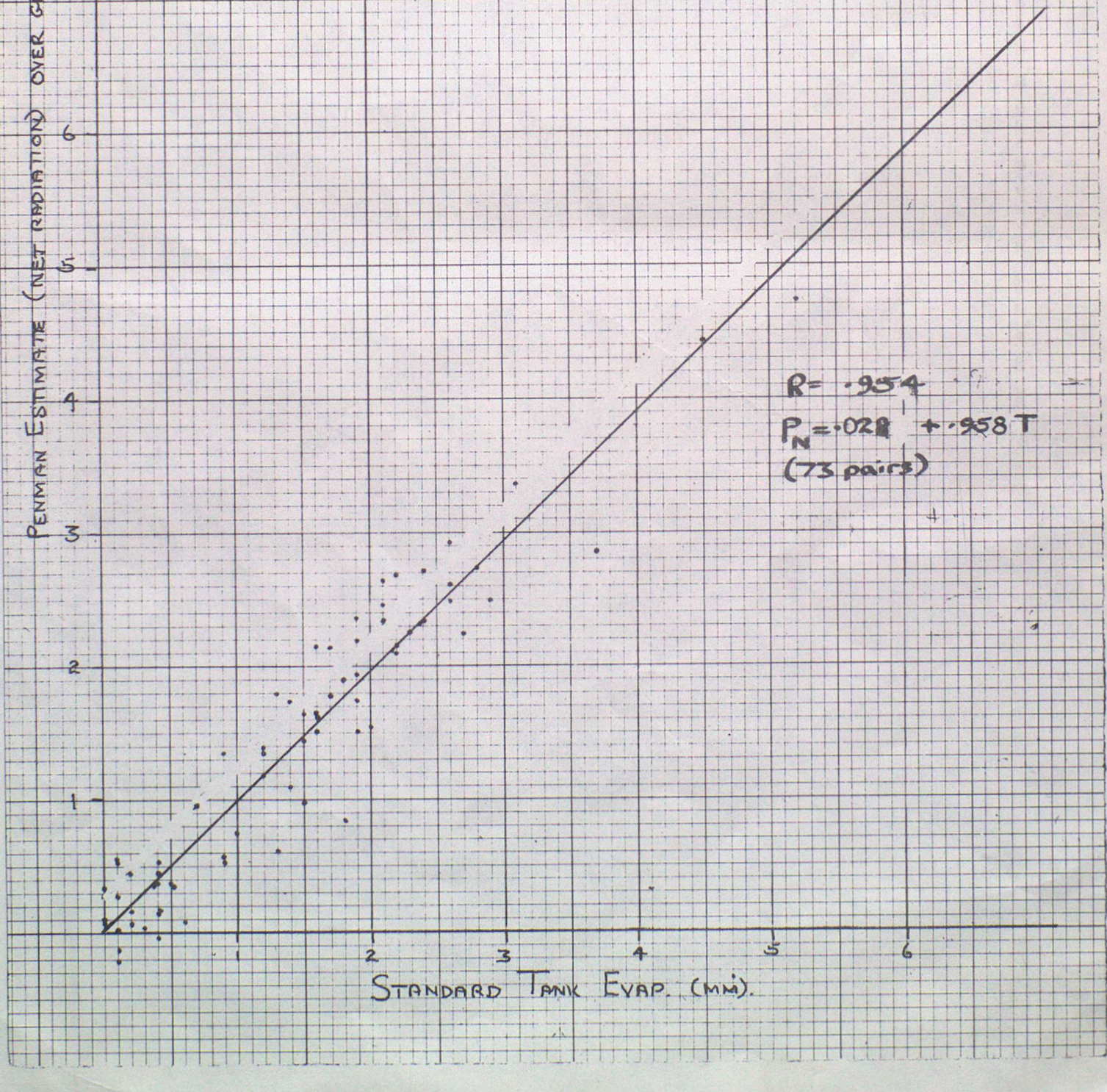


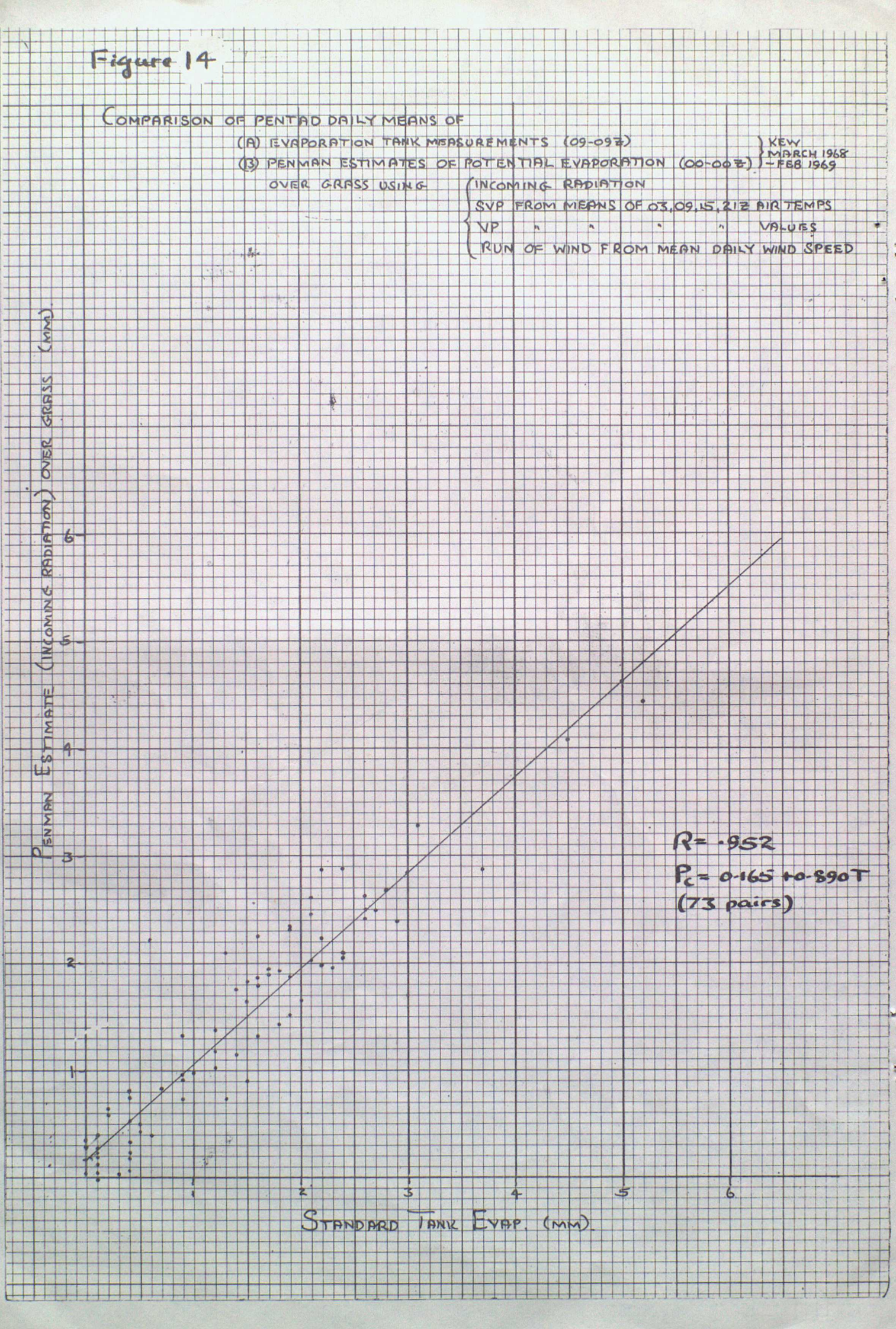
Figure 14

COMPARISON OF PENTAD DAILY MEANS OF

(A) EVAPORATION TANK MEASUREMENTS (09-09Z) } KEW MARCH 1968 – FEB 1969

(B) PENMAN ESTIMATES OF POTENTIAL EVAPORATION (00-00Z) } KEW MARCH 1968 – FEB 1969

OVER GRASS USING { INCOMING RADIATION; SVP FROM MEANS OF 03, 09, 15, 21Z AIR TEMPS; VP " " " " VALUES; RUN OF WIND FROM MEAN DAILY WIND SPEED



VIA

Figure 15

COMPARISON OF PENTAD DAILY MEANS OF

(A) EVAPORATION TANK MEASUREMENTS (09-09 Z) } KEW MARCH 1968 FEB 1969

(B) PENMAN ESTIMATES OF POTENTIAL EVAPORATION (09-09Z) } KEW MARCH 1968 FEB 1969

OVER GRASS USING
- ALBEDO ·25
- SUNSHINE
- SVP FROM MEANS OF 03, 09, 15, 21 Z AIR TEMPS
- VP FROM " " " " VALUES
- RUN OF WIND FROM MEAN DAILY WIND SPEED.

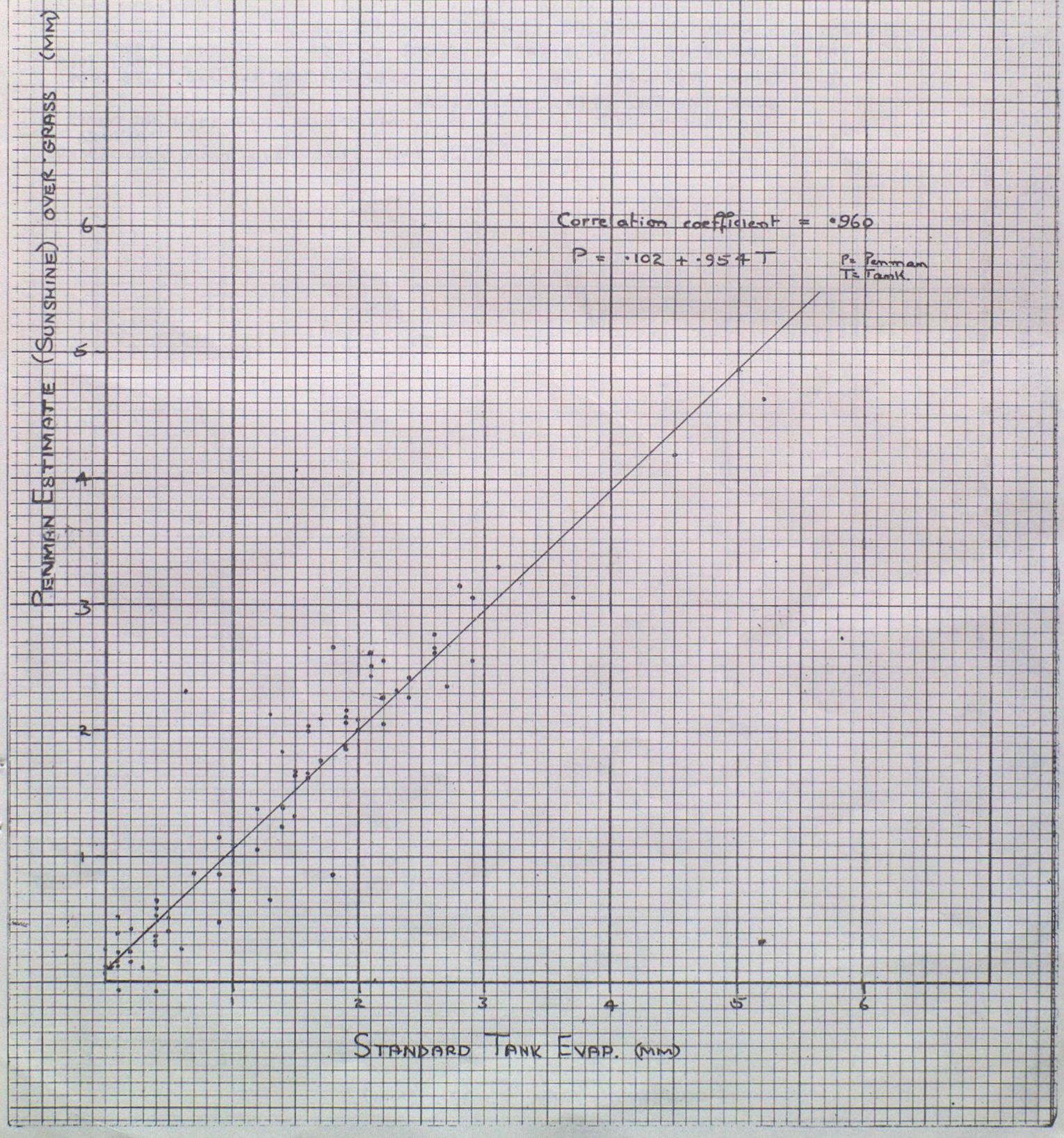


Figure 16

COMPARISON OF DAILY MEANS OF PENTAD VALUES.

(A) EVAPORATION TANK MEASUREMENTS 09-09Z } KEW MARCH 1968 - FEB 1969.

(B) PENMAN ESTIMATES OF POTENTIAL EVAPORATION 00-00Z }

OVER WATER USING ALBEDO .07

- SUNSHINE
- SVP FROM MEANS OF 03, 09, 15 & 21 AIR TEMPS
- VP " " " " VALUES
- RUN OF WIND FROM MEAN DAILY WIND SPEED.

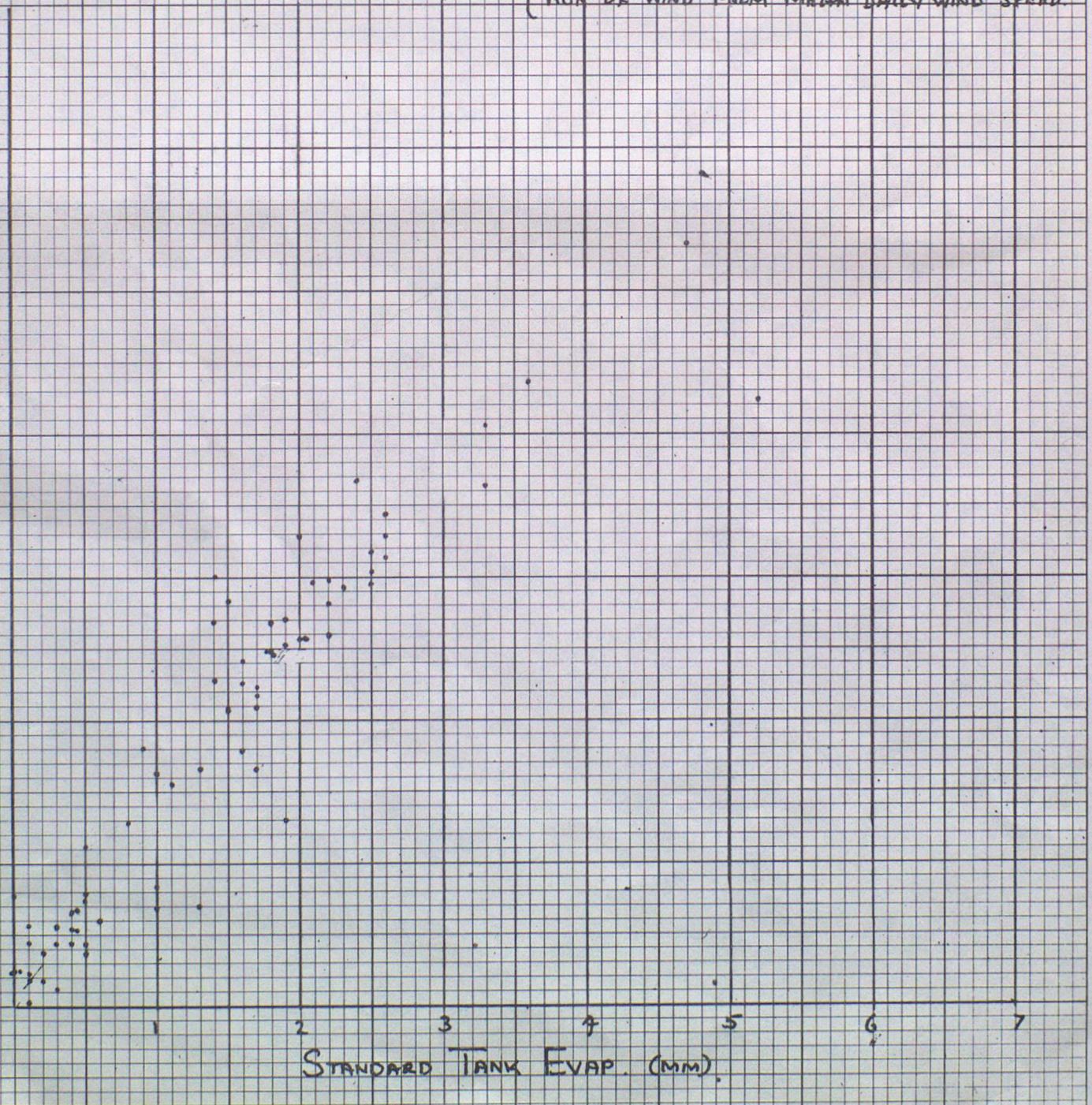
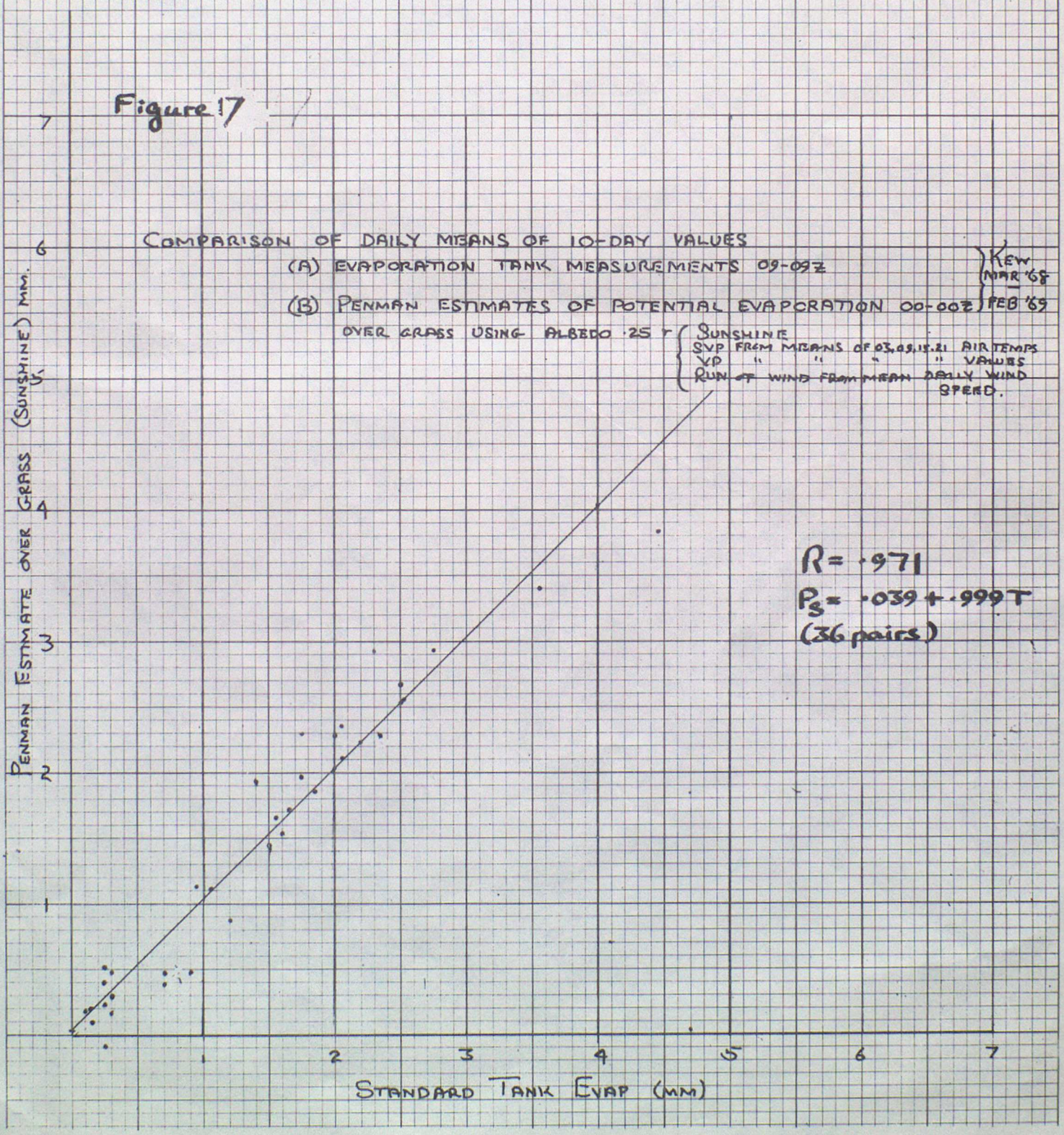


Figure 17


COMPARISON OF DAILY MEANS OF 10-DAY VALUES
(A) EVAPORATION TANK MEASUREMENTS 09-09Z } KEW MAR '68
(B) PENMAN ESTIMATES OF POTENTIAL EVAPORATION 00-00Z } FEB '69
OVER GRASS USING ALBEDO ·25 T { SUNSHINE, SVP FROM MEANS OF 03,09,15,21 AIR TEMPS, VP " " " " VALUES, RUN OF WIND FROM MEAN DAILY WIND SPEED.
R = ·971
Ps = ·039 + ·999T
(36 pairs)
PENMAN ESTIMATE OVER GRASS (SUNSHINE) MM.
STANDARD TANK EVAP (MM)
1
2
3
4
5
6
7

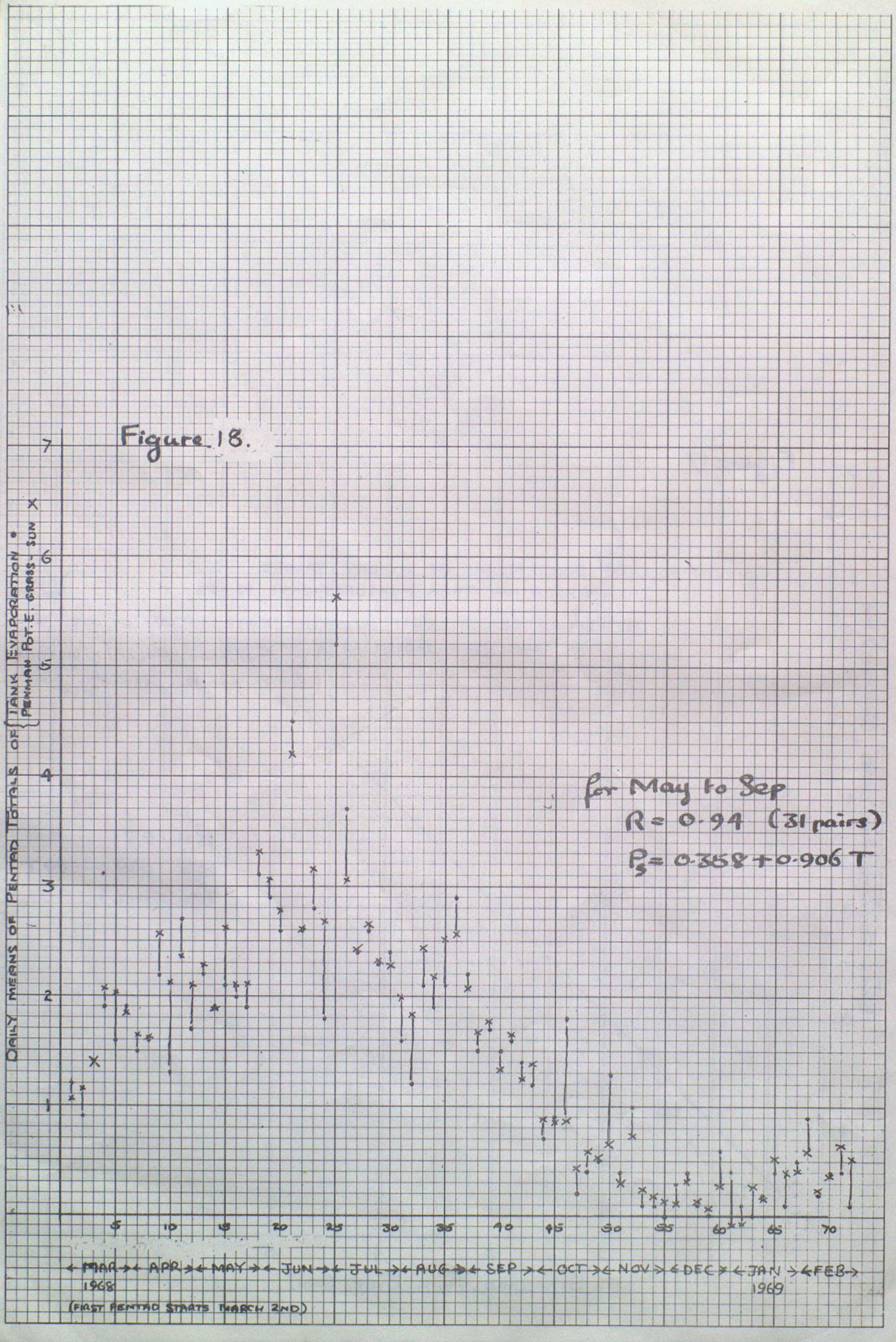


Figure 18.
DAILY MEANS OF PENTAD TOTALS OF { TANK EVAPORATION • PENMAN POT. E. GRASS - SUN X
for May to Sep
R = 0.94 (31 pairs)
P_s = 0.358 + 0.906 T
1
2
3
4
5
6
7
5
10
15
20
25
30
35
40
45
50
55
60
65
70
← MAR → ← APR → ← MAY → ← JUN → ← JUL → ← AUG → ← SEP → ← OCT → ← NOV → ← DEC → ← JAN → ← FEB →
1968
1969
(FIRST PENTAD STARTS MARCH 2ND)

PENTAD PE ESTIMATES FOR KEW
(SUNSHINE — ALBEDO 0·25)
COMPARED WITH PENTAD PE ESTIMATES
UP-DATED BY TANK EVAPORATION.

Fig. 19

IN EACH SET OF POINTS
CROSSES DENOTE $\overset{5}{\underset{1}{P}}, \overset{6}{\underset{2}{P}}, \overset{7}{\underset{3}{P}}, \overset{8}{\underset{4}{P}}, \overset{9}{\underset{5}{P}}, \overset{10}{\underset{6}{P}}$

DOTS DENOTE $\overset{5}{\underset{1}{P}} - T_1 + T_6$, $\overset{5}{\underset{1}{P}} - (T_1+T_2) + (T_6+T_7)$, $\overset{5}{\underset{1}{P}} - (T_1+T_2+T_3) + (T_6+T_7+T_8)$,

$\overset{5}{\underset{1}{P}} - (T_1+T_2+T_3+T_4) + (T_6+T_7+T_8+T_9)$,

$\overset{5}{\underset{1}{P}} - (T_1+T_2+T_3+T_4+T_5) + (T_6+T_7+T_8+T_9+T_{10})$

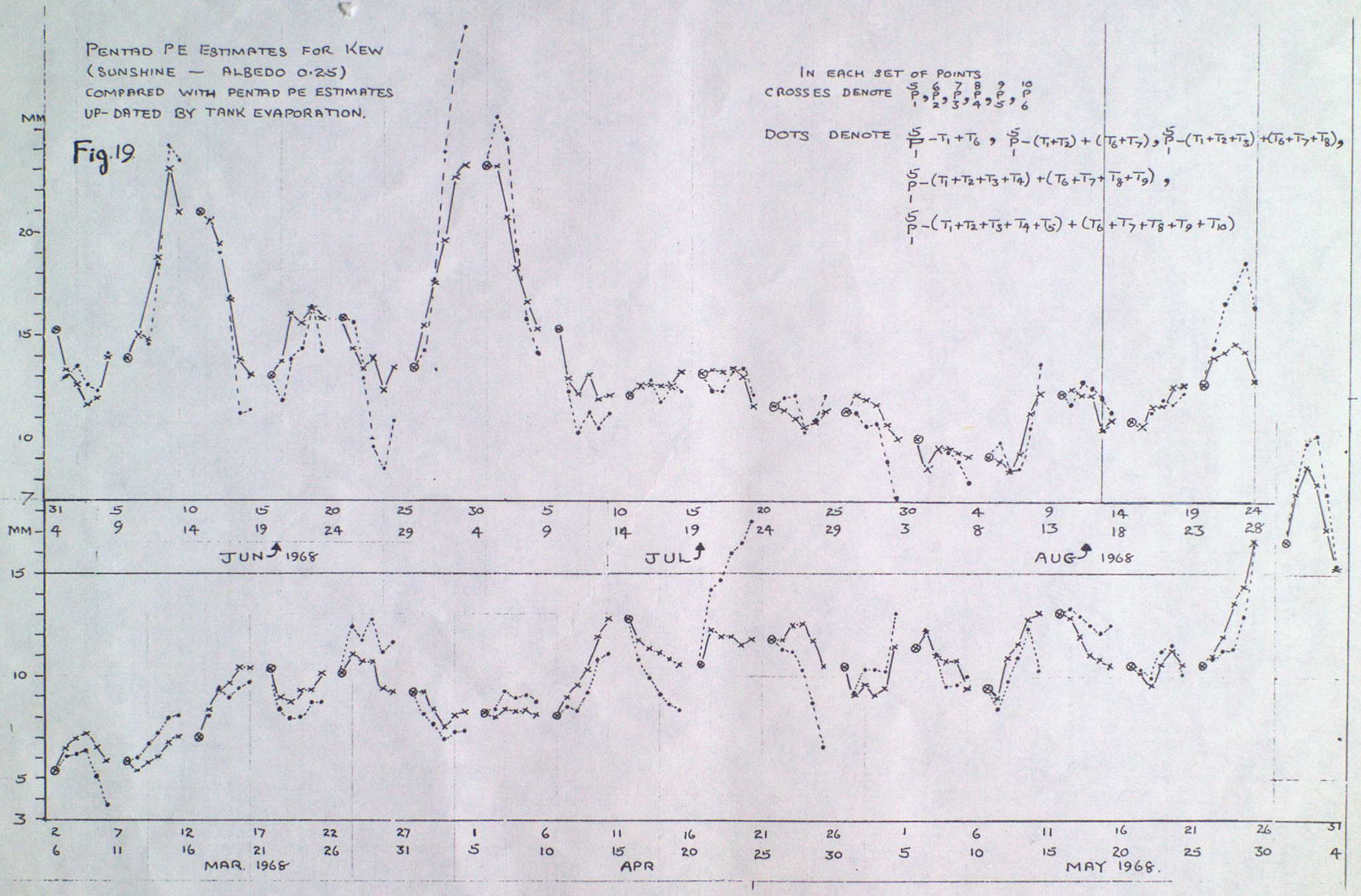


Fig. 20

KEW MAR 1968 - FEB 1969 (SUNSHINE - ALBEDO .25).

CROSSES ARE $\overset{5}{P}_1$ CONSECUTIVE PE ESTIMATES.

DOTS $\overset{5}{P}_1 - \overset{5}{T}_1 + \overset{10}{T}_6 = \overset{10}{P}_6$

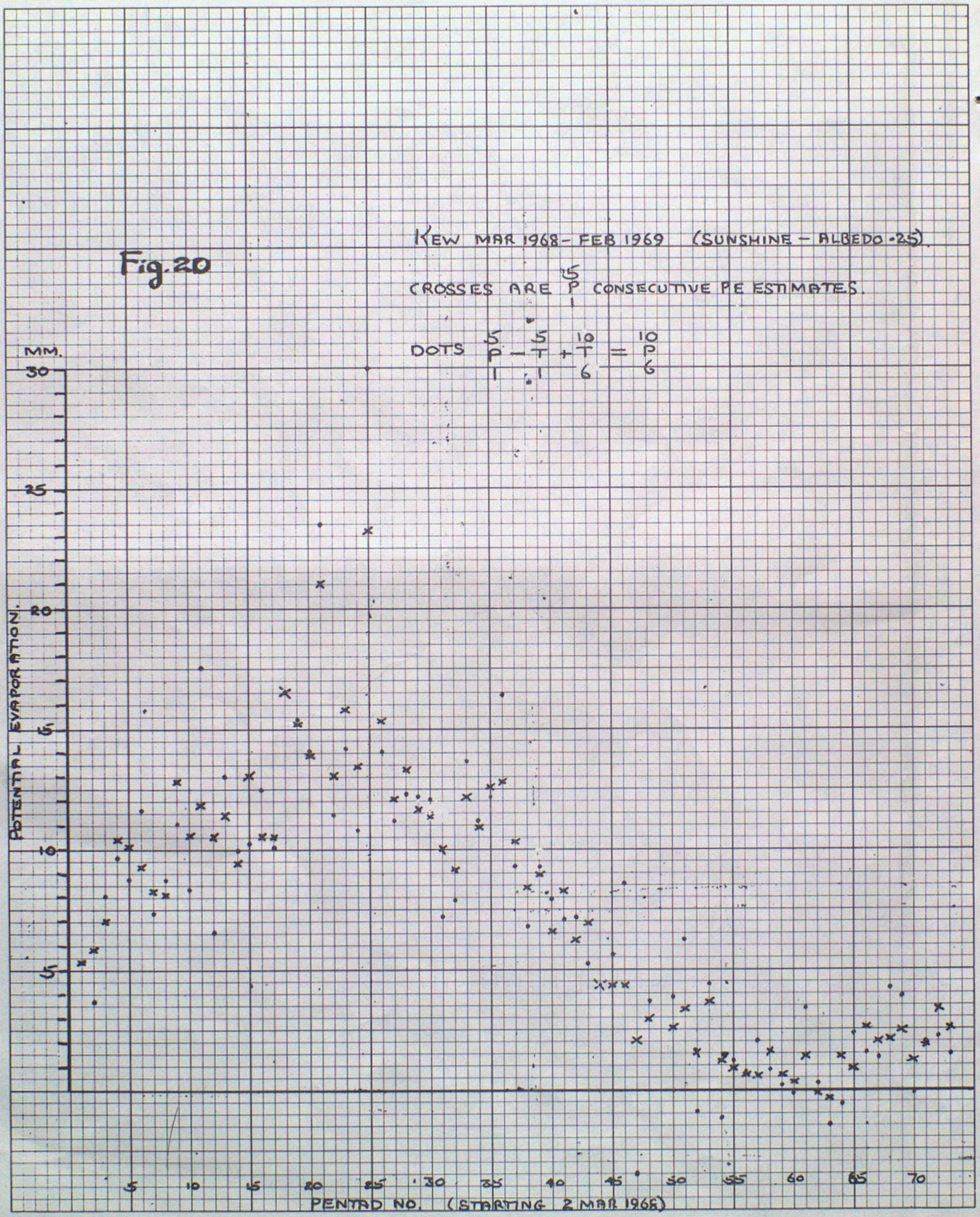


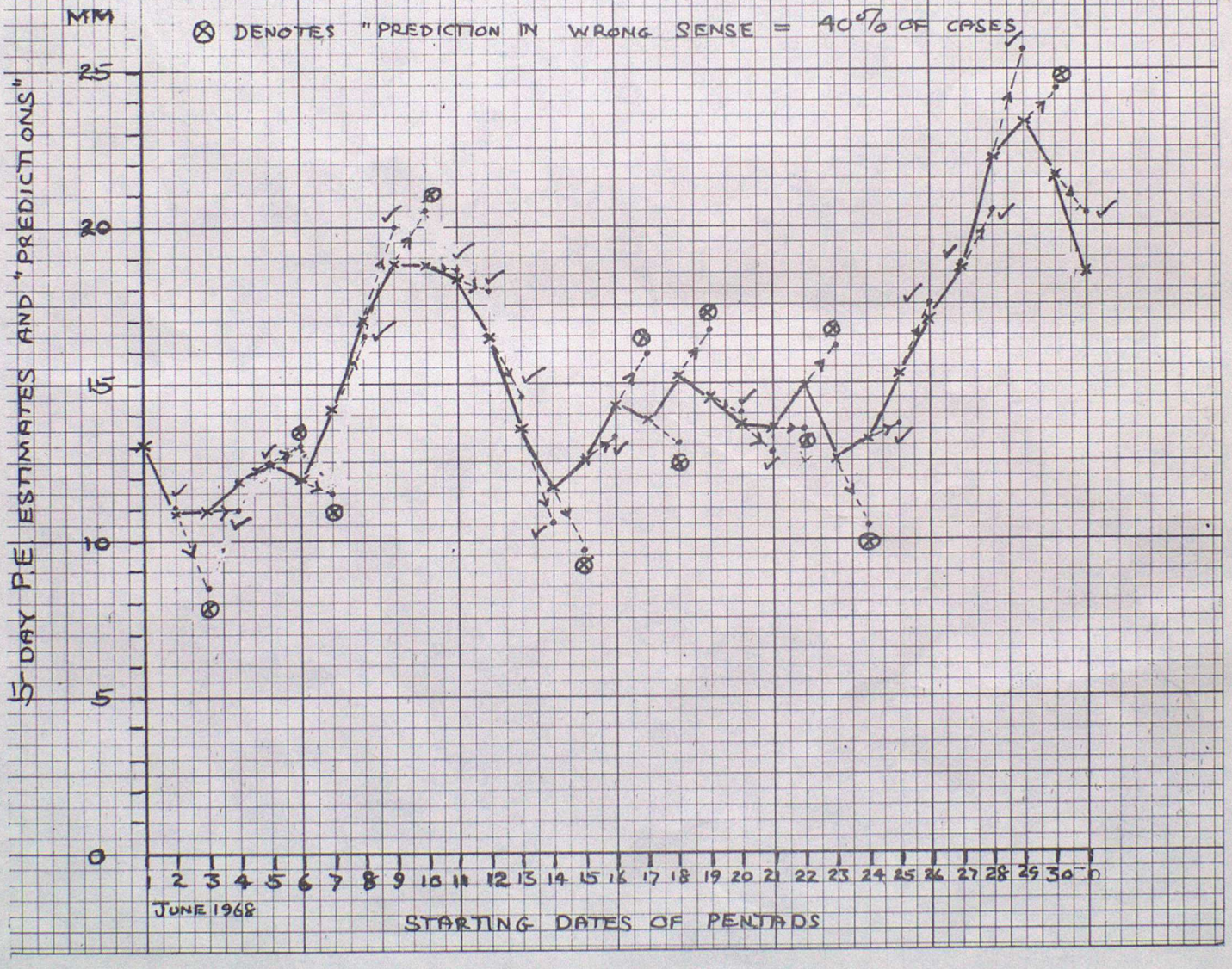
Fig. 21

MILDENHALL.

POINTS PLOTTED X——X ARE 5-DAY TOTALS OF P.E. (ON STARTING DATE OF PENTAD)
POINTS PLOTTED • AND JOINED TO X-POINTS BY DASHED LINES ARE "PREDICTIONS"
OF NEXT 5-DAY TOTAL OF P.E. (X-POINT) FORMED AS FOLLOWS :

$$\sum_{\text{day 2}}^{\text{day 6}} PE = \sum_{\text{day 1}}^{\text{day 5}} PE - PE_{\text{day 1}} + PE_{\text{day 6}}$$

WHERE $\sum$ DENOTES A 5-DAY ESTIMATE
NOT THE SUM OF 5 ESTIMATES.

✓ DENOTES "PREDICTION IN CORRECT SENSE = 60% OF CASES

⊗ DENOTES "PREDICTION IN WRONG SENSE = 40% OF CASES